Formula Sheet for the second Midterm Exam

1. Let $T_n(x)$ be the Taylor polynomial centered at a for f. Then

$$R_n(x) = f(x) - T_n(x) = \frac{1}{n!}\int_a^x (x-u)^n f^{(n+1)}(u)\,du.$$

2. Suppose that $\{a_n\}$ and $\{b_n\}$ are positive sequences.

(a) If $\sum_1^\infty b_n$ converges and $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} < \infty$, then $\sum_1^\infty a_n$ converges.

(b) If $\sum_1^\infty b_n$ diverges and $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} > 0$, then $\sum_1^\infty a_n$ diverges.

3. Arc length of a parametric curve $(x(t), y(t))$, $a \le t \le b$:

$$s = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\,dt.$$

4. Area of sector between $r = f_1(\theta)$, $r = f_2(\theta)$, $\theta = \theta_1$ and $\theta = \theta_2$:

$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} \left[f_2^2(\theta) - f_1^2(\theta)\right]\,d\theta.$$

5. (a) $\int_1^\infty \frac{1}{x^p}\,dx$ converges if $p > 1$ and diverges if $p \le 1$.

(b) If $b > 0$, $\int_0^b \frac{1}{x^p}\,dx$ diverges if $p \ge 1$ and converges if $p < 1$.

6. Let f be a decreasing, positive function defined for $1 \le x < \infty$, and let $a_n = f(n)$. Then

$$0 < \sum_{n=1}^{\infty} a_n - \sum_{1}^{N} a_n = \sum_{N+1}^{\infty} a_n \le \int_N^\infty f(x)\,dx.$$