Review Problems for the first Midterm

1. Determine whether the improper integral converges and, if so, evaluate it.

$$a) \quad \int_1^\infty \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx \qquad b) \quad \int_1^2 \frac{1}{x\ln x}\,dx \qquad c) \quad \int_1^\infty xe^{-2x}\,dx$$

2. Use the comparison test to determine whether the integral converges.

$$a) \quad \int_1^\infty \frac{1}{\sqrt{x^7+2x+1}}\,dx \qquad b) \quad \int_1^\infty \frac{1}{x^{1/3}+x^{2/3}}\,dx \qquad c) \quad \int_1^2 \frac{e^x}{x}\,dx$$

3. Find the (x, y) coordinates of the points on the curve defined by $(x(t), y(t)) = (t + \sin t, t - 2\sin t)$, $0 \leq t < 2\pi)$ where the tangent is horizontal, and also where the tangent is vertical.

It t represents time in seconds and $(x(t), y(t))$ represents the position of a particale in units of meters, how fast is the particle moving when $t = \pi/2$?

4. Show that the locus of the polar equation $r = 5\sin\theta$ is a circle and find the equation of that circle in cartesian coordinates.

5. Write the equation $y = x^2$ defining a parabola in polar coordinates. Use the polar coordinate equation to find the area of the region bounded by the line L and the parabola $y = x^2$, wher L is the line through the origin making an angle of $\theta = \pi/12$ with the x-axis.

6. Problem 11.5 in the text.

7. Find the Taylor polynomial $T_3(x)$ centered at $a = 0$ of $f(x) = (1 + x)^{5/4}$. Use the remainder formula to determine the sign of $f(-0.1) - T_3(-0.1)$. Find an upper bound on the error $|f(x) - T_3(x)|$ valid for all x such that $-0.2 \leq x \leq 0.2$.

Chapter 10 review exercises, pages 617-620. 9, 13, 16 (Hint: Write $\ln(n + 1) = \ln[(1 + 1/n)n]$), 21, 30, 31, 34.

Do also the following using whatever method you like to determine convergence or divergence; if the series has positive and negative terms, determine whether it is absolutely convergent, conditionally convergent): 35, 36, 41, 43, 44, 45, 51, 53, 61, 59, 63, 67, 71, 73, 75.

For problem 44, also find an N so that the partial sum s_N approximates the infinite series to an accuracy of 10^{-3}. (Use the error bound from the workshop problem and bound the function $x/(x^5 + 5)^{1/2}$ above by another function that is easy to integrate.)

For the test you will be given a formula sheet with integral tables, error bound and remainder formula for Taylor polynomials, and integral error bounds for approximation of infinite series with positive terms by partial sums.