Review problems for the first midterm

1. Exercises 37 and 38 in Chapter Review Exercises of Chapter 6 (pp. 414-415. For these problems also set up definite integrals for the volume obtained by rotating the given region about the axes $y = -2$ and $x = 3c$. (Don't evaluate the integrals.)

2. Exercise 41, Chapter 6 review exercises (pp. 414-415).

3. Problem 28, section 6.5.

4. The base of a solid is bounded by the curve $y = x^2$ and the line $y = 4$. Cross sections of the solid by planes perpendicular to the y-axis are squares. Find the volume of the solid.

5. How large must n be in order that the midpoint method approximates $\int_1^3 \sin(\ln x)\,dx$ to an accuracy of 10^{-4}? (In this problem obtain a rough bound by hand (that is, not by graphing on the calculator) for the second derivative of the integrand by hand. Remember the triangle inequality $|a + b| \le |a| + |b|$.)

6. Write the correct form (with undetermined constants) of the partial fraction decomposition of the rational function

$$\frac{x^5 + 5x^3 - 2}{(x+2)(x-4)^2(x^2+3x+5)^2}.$$

DO NOT determine the constants.

7. Evaluate the following integrals using only the formula sheet:

a) $\displaystyle\int \frac{dt}{t^2 + 4t - 5}$

b) $\displaystyle\int \frac{x\,dx}{x^2 + 4x + 10}$

c) $\displaystyle\int x^2 e^{4x}\,dx$

d) $\displaystyle\int \cos^9(6\theta)\sin^3(6\theta)\,d\theta$

e) $\displaystyle\int_0^{\pi/2} \sin(2x)\sin^2(x)\,dx.$

f) $\displaystyle\int e^x \cos(2x)\,dx$

g) $\displaystyle\int \frac{(\ln x)^2}{x^2}\,dx$ (Hint: First use substitution and then integration by parts.)

h) $\displaystyle\int \frac{x^2}{(x-2)(x^2+25)}\,dx$

i) $\displaystyle\int \frac{dz}{z^3\sqrt{z^2-1}}.$

j) $\displaystyle\int \frac{\cos(\theta)}{1-(\sin(\theta))^2}\,d\theta.$

k) $\displaystyle\int \frac{1}{x+3\sqrt{x}+2}\,dx.$

l) Show $\displaystyle\sec^3 x\,dx = \tan x \sec x - \int \tan^2 x \sec x\,dx.$ Integrate by parts. Use this to show that $\displaystyle\int \sec^3\,dx = (1/2)[\tan x \sec x + \ln|\sec x + \tan x|\,] + c.$