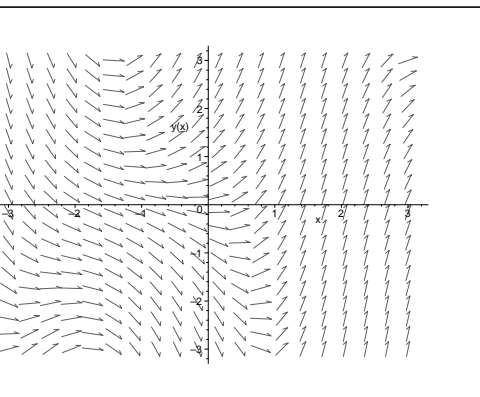


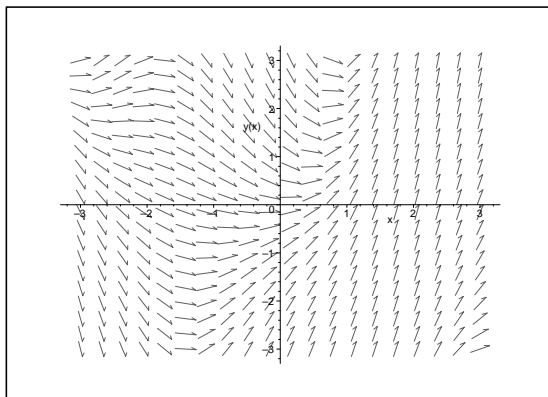
Review Exam I — Mathematics 244

1. Consider the differential equation $y' - \cos(x)y = xe^{\sin(x)}$.

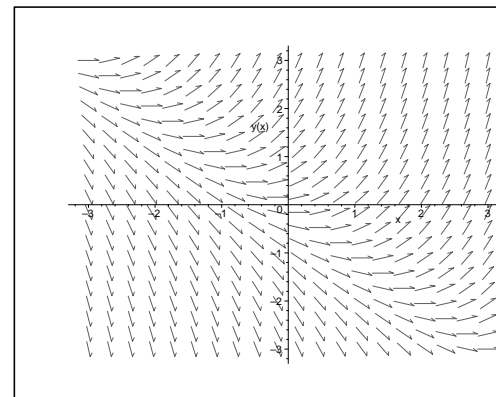
a) Which figure below is the direction field for this equation? Explain.



a



b



c

b) Find the general solution to this differential equation.

c) Do the solutions of this differential equation remain bounded as $x \rightarrow \infty$? Explain your reasoning.

2. Consider the differential equation $(x \sin 2y + \log x)dx + (x^2 \cos 2y + y^2)dy = 0$.

a) Show that this equation is exact.

b) Find an equation which implicitly describes all solutions of the differential equation.

c) A function $\phi(x)$ satisfies $\phi(1) = 0$, $\phi(\pi) = 1$. Can such a function solve the differential equation? Why or why not?

3. Consider the differential equation $y'' - 5y' + 6y = 0$

a) Find fundamental solutions y_1, y_2 of this differential equation.

b) Compute the Wronskian of your functions y_1, y_2 .

c) Find a solution of $y'' - 5y' + 6y = 0$, $y(0) = -1$, $y'(0) = 1$. Is it unique? Explain.

4. Solve the differential equation

$$\frac{dy}{dx} = \frac{x + y}{x - y}.$$

5. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. Suppose freshly poured coffee has a temperature of 200° F, and 1 minute later its temperature is 190° F in a 70° F room.

a) Set up a differential equation and initial value problem that describes the temperature $\theta(t)$ of the coffee as a function of time t .

b) Find a formula for the temperature $\theta(t)$ of the coffee at time t .

c) How long does it take for the coffee to cool to 150° F?