

Review Exam II — Mathematics 244, Fall 2001

The following problems are generally similar to the type of problems you can expect to find on the exam. However, not all types of problems are included and it is possible that the exam will contain some problems quite different from any here.

1. Consider the initial value problem

$$y'' + 4y = 5xe^x, y(0) = 2, y'(0) = -1.$$

Find the solution to this equation. What is the value of the solution at $x = \pi$? How does the solution behave as $x \rightarrow \infty$?

2. Consider the differential equation

$$x^2 y'' - 2y = 3x^2 - 1, x > 0$$

Find the general solution to this equation. Find the solution $y(x)$ which satisfies $y(1) = 1/6, y'(1) = 4/3$. How does this solution behave as x decreases from 1 to 0?

3. Let γ be a nonnegative real number and consider the differential equation describing the motion of a spring and mass system:

$$x'' + 2\gamma x' + 169x = 0, x(0) = 0, x'(0) = 6$$

When $\gamma = 0$ what is the frequency and amplitude of the spring motion? For which γ is the solution critically damped?

4. Find the general solution to

$$y^{iv} - 8y' = 0.$$

5. Find all eigenvalues of the matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ 4 & 4 & 4 \\ -6 & 1 & -5 \end{pmatrix}$$

For each eigenvalue, find a corresponding eigenvector.

6. Solve the initial value problem for $\mathbf{x}(t)$: $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, where

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 5 & 6 \\ -1 & -3 & -4 \end{pmatrix}$$

7. Match the functions below with the differential equations that they satisfy. Use the theory and methods developed in the course to efficiently carry this out.

a) $e^{3x} + e^{-x}$

b) $x^4 + x^{-2}$

c) $\sin(2x) + \cos(2x)$

d) $xe^x + x + 2$

1) $x^2y'' - xy' - 8y = 0$

2) $y'' + 4y = 0$

3) $y'' - 4y = 0$

4) $y''' - y'' + 4y' - 4y = 8$

5) $y'' - 2y' + y = x$