Mathematics 244: Lab 0   INTRODUCTION TO MAPLE FOR DIFFERENTIAL EQUATIONS

This lab is intended to introduce you to some of the features of Maple that are useful in solving differential equations and to give you practice preparing a Maple worksheet. Other resources can be found on the web page for Math 244.

Although this lab is for practice only, it is important that you learn how to prepare your Maple worksheet in the way you will be asked to do it in future assignments on which you will be graded. All projects ask you to interpret the results obtained by Maple. You should use the **text** feature of Maple to insert your observations into the worksheet (don't write them by hand). Graphs should be generated using the default "inline" option, so that they appear in your worksheet. The **title** option should be used to include a brief description with each graph. The final worksheet should be **edited** to remove any extraneous material such as any errors you have made or the reminder at the beginning of the seed file to add your personal header.

Earlier versions of these lab descriptions included samples of the Maple instructions used. These snippets of Maple were also available as a "seed file", downloadable from the web page. The ease of obtaining the seed file means that verbatim quotes of Maple instructions in the descriptions can be phased out. The seed file uses an outline structure to mimic the organization of the lab description. This outline should be fully expanded when you print your final copy.

0. Setup. In order to use some of the special commands Maple has for producing plots and solving differential equations, you must first type:

```
with(plots):  with(DEtools):
```

1. Expressions, derivatives and graphs. The seed file contains instructions for producing the Maple expression `f0` representing $e^{\sin x}$ and expressions `f1` and `f2` for its first two derivatives. These three quantities are then graphed on the interval $-\pi \le x \le \pi$, using red for the function, green for the first derivative, and blue for the second derivative. (Note that the instructions giving names to the separate plots end with colons rather than semicolons to hide the details of the plot structure while the `display` command ends with a semicolon to show the plot. The use of the `title` option is also illustrated here.)

Construct expressions `g0`, `g1` and `g2` for the expression $\sin(x^2)\ln(x)$ and its first two derivatives. Then, plot these three expressions on the same set of axes over the interval $0.1 \le x \le 2$, using red for the given expression, green for the first derivative, and blue for the second derivative.

Discussion. Why do only positive values of x appear in the g expressions? What happens as $x \to 0$? (Use you knowledge of Calculus to study the limits, and then experiment with different x intervals. Replace the *reminder* with a summary of your observations and remove any experiments.)

2. . Consider the expression

$$2\ln F - 1.2F + \ln R - 0.9R.$$

Because the expression includes $\ln R$ and $\ln F$, it is only defined for $R > 0$ and $F > 0$. To get an idea of the behavior of the function, it can be graphed. The following instructions introduce a name for the expression and a graph showing contours of the expression.

```
ex2:=2*ln(F)-1.2*F+ln(R)-0.9*R;
contourplot(ex2,R=0.5..3,F=0.25..3,title="R versus F");
```

Discussion. The value enclosed by the contours is a local maximum (actually a **global** maximum). Find the location of the maximum (find the derivatives using Maple or by hand calculation, whichever you prefer). If one of the variables is fixed, what happens to the function as the other variable approaches zero? (You may want to experiment with different (R, F) ranges in the `contourplot` instruction to see the effect on the contours that are automatically selected, but such experiments should be removed from the worksheet even if they influence your discussion.)

3. . Consider the expression

$$ex3 = \frac{y^2}{2} + e^y - \frac{x^2}{2} - e^{-x}$$

When $x = 0$ and $y = 1$, the value of the expression $ex3$ is $e - 1/2$.

For each value of C, this defines y implicitly as a function of x. If we are interested in such a function with $y(0) = 1$, then we substitute $x = 0$ and $y = 1$ into the equation to get $C = e - 1/2$. Note the form in which Maple gives this answer and see if you can convert it to this form.

Then, use the `contourplot` command to obtain a plot of a family of solutions over the range $-2 \leq x \leq 4$, $0 \leq y \leq 4$. (The instruction is contained in the seed file.)

Discussion. The restriction to positive values of y has the effect of giving **at most one** value of y for each choice of C and x. Describe how to show that the expression $ex3$ is an **increasing** function of y when x is fixed, and the relevance of this fact to the ability to solve $ex3 = C$ for y. Then, find a value of C for which this solution cannot be defined as a function of x for all x. Illustrate by using Maple to produce an `implicitplot` of `ex3=C` for this value of C.

4. Suppose a ball of mass m is throw upward from a height h with initial velocity v. If the only force acting is gravity, then Newton's second law of motion says that the mass satisfies the differential equation

$$m\frac{d^2y}{dt^2} = -mg,$$

where g is the acceleration due to gravity and $y(t)$ is the height of the ball above the ground at time t. Note that y satisfies the initial conditions $y(0) = h$ and $y'(0) = v$. We now show how Maple can be used to find a formula for $y(t)$. The instructions

```
de4:= diff(y(t),t,t) = -g;
ivp4:={de4,y(0)=h,D(y)(0)=v};
ans4:=dsolve(ivp4,y(t));
```

are in the seed file. This requires that the special package for solving differential equations has been previously loaded, but we did that at the start of the worksheet. The first statement defines the differential equation, giving it the name `de4` (note the common factor m has been removed). Also note that in the use of `diff`, the function y must be referred to as $y(t)$. The second statement defines an initial value problem, consisting of the differential equation and two initial conditions. Note the use of `D` (which stands for derivative) to define the initial condition $y'(0) = v$. The last statement uses the Maple command `dsolve` to solve the initial value problem for the unknown function $y(t)$. The result of this solution is saved under the name `ans4`.

The result, `ans4`, of the `dsolve` command is an equation. If we want to verify that the result produced by Maple really is a solution of the differential equation, we can write

```
subs(ans4,de4);
eval(%);
```

to see the result of substituting this $y(t)$ into the equation and evaluating the result. Alternatively, we can use the `rhs` command to obtain an **expression** for the solution $y(t)$. We then use the `diff` and `subs` commands to check that this expression satisfies the differential equation and the initial conditions. The following instruction are in the seed file to carry out these steps.

```
sol4:=rhs(ans4));
sol4t:=diff(sol4,t);
sol4tt:=diff(sol4t,t);
sol4tt+g;
subs(t=0,sol4);
subs(t=0,sol4t);
```

Discussion. Why do these computations check that you have a solution to the initial value problem? Give a brief comment to indicate that each property has been verified. Since Maple provides an interactive environment, it is possible to perform verifications without translating everything into the Maple language. Comments allow you to explain the connection between the worksheet and the problem being studied.

5. An example from the textbook. Exercise 24 in section 1.1 asks you to study the differential equation

$$\frac{dy}{dt} = 2t - 1 - y^2.$$

In particular, you are asked to produce a slope field and use this to determine the relation between an initial condition at $t = 0$ and the behavior as $t \to \infty$. The following instructions show some solutions in a modest region of the plane that can suggests the long-term behavior of solutions. It also indicates that Maple has a formula for the solution. Describe the likely long-term behavior based on the graph. Then use Maple Help to interpret the explicit solution.