Mathematics 244: Lab 2

Fall 2003

0. Introduction and Setup. In this lab, we shall use Maple's ability to plot direction fields and approximate the solution of differential equations by numerical methods to understand the solutions of differential equations.

Please turn in only the printout of your Maple worksheet. Include explicit answers to all questions asked, using the **text** feature of Maple to insert them in the worksheet. Remove from the worksheet any extraneous material and any errors you have made.

Before you start up Maple, copy the seed file into your directory from the Web page of the course. After you start up Maple, import this file into Maple by choosing **Open** from the **File** menu as described in the handout **Instructions for Use of Maple in Mathematics 244**. You will be asked to execute the Maple commands that have been placed in this worksheet and to add some of your own.

As usual, the seed file begins by loading the `plots` and `DEtools` libraries. Several plots are produced in these exercise. **Be sure to supply tiles for each plot that is shown in your worksheet.** (Titles are not shown in this description, but plot command in the seed file include titles.)

1a. Obtaining graphs for first equation. In Lab 0 and Lab 1, we saw how the direction field of a differential equation could be obtained by use of the command `DEplot`. In cases where the exact solution of an initial value problem can be obtained, this solution may be plotted together with the direction field by combining the `DEplot`, `plot` and `display` commands. This serves as a check of your solution since it should detect if the slope field does not appear tangent to the solution curves.

A more direct approach gives initial conditions to the `DEplot` command. In this case, Maple computes a **numerical** solution to a differential equation and then plots the resulting solution curve together with the direction field of the differential equation.

The sequence of commands described in Problem 1a have already been entered into the seed file. Execute them to see how Maple can be used to plot the direction field along with the particular solution satisfying this differential equation and the initial condition $y(0) = 0$.

```
de1:= diff(y(x),x) = 2 + 3*x*y(x)
initval1a:={[y(0)=0]};
DEplot(de1, y(x), x=-3..3, initval1a,y=-3..3);
```

The `DEplot` command has many options which you can read about in the Maple help pages. In particular, it is possible to specify the color of the solution curve by using `linecolor`, the color of the direction field by using `color`, to make the gridwork of lines in the direction field finer by using the `dirgrid` option, and to get a finer resolution in computing the solution with the `stepsize` option. For example, try:

```
opt1:=linecolor = black,color = green, dirgrid= [30,30],stepsize=0.1
DEplot(de1, y(x), x=-3..3, initval1a, y=-3..3, opt1);
```

1b. Plotting several solution curves. It is also possible to plot several solution curves on the same graph. For example, type

```
initval1b:= {[y(0)=-2],[y(0)=-1],[y(0)=1],[y(0)=2]};
DEplot(de1, y(x), x=-3..3, initval1b, y=-3..3,opt1);
```

1c. Classifying solutions graphically. By varying the initial conditions, it is possible to approximate special solutions. For example, the solution of this equation with $y(0) = -1$ is everywhere increasing, but the solution with $y(0) = -2$ reaches a maximum value and then becomes rapidly decreasing as $x \to +\infty$. By plotting solutions with other values of $y(0)$ between -2 and -1, you can identify more precisely where this change of behavior occurs. **Produce a graph** with values of $y(0)$ separated by 0.1 containing **exactly two solutions** of each type.

1d. Discussion. The graph suggest some properties that can be investigated by looking more closely at the differential equation. Apart from suggesting these questions, Maple may not be needed in finding their answers.

In the quadrant where $x > 0$ and $y > 0$, solutions seem to be steadily increasing. How can this be proved from the differential equation?

The other solution seem to have a unique maximum point. Find the curve containing the maximum points on all solutions in the quadrant where $x > 0$ and $y < 0$.

2a. First graph of second equation. Consider the differential equation

$$\frac{dy}{dt} = \left(2t^2 - y^2\right)\sin y$$

with the initial condition $y(0) = -1$.

Introduce the name `de2` for this equation and use the `DEplot` command to plot a direction field and the (numerical) solution of this initial value problem for $-3 \leq t \leq 6$ and $-4 \leq y \leq 0$ with **no special options**. The result will not be satisfactory, and you should not try for a better graph.

2b. Improving the graph. Now introduce the options

```
opt2:=linecolor = black,color = green, dirgrid= [24,12],stepsize=0.05
```

and add `opt2` to the `DEplot` command to get an improved plot. Your worksheet should contain both the original and improved plots, suitably titled.

2c. Discussion. Discuss the properties of the equation that require that the solution of this initial value problem have $y(t) > \pi$ for all t. How does this help recognize that the first graph is inaccurate?

3a. Forcing Euler's Method. Maple help claims that the `DEplot` command uses the classical fourth order Runge-Kutta method (as described in Section 8.3) for producing numerical solutions to initial value problems. For the interval $[-3, 6]$, the standard step size is 0.45, and our option improved it to 0.05.

It is possible to use other methods for the numerical work. While Euler's method is **less accurate**, its simplicity will allow a clearer picture of the cause of the unsatisfactory behavior of the solution.

To use Euler's method (with the standard step size of 0.45), introduce the option

```
opt3:=method=classical[foreuler];
```

Then give the `DEplot` command for the initial value problem described in 2, with this option (and no other options).

3b. Other step sizes. Produce graphs with a larger step size of 0.6 as well as the smaller step sizes 0.3 and 0.15.

3c. Discussion. What does the graph using Euler's method reveal about the cause of the inaccuracy? Compare results with the default step size of 0.45 with the other step sizes user in 3b. How much of the graph seems accurate for each of these step sizes.