
Math 244 Sample Exam 1

September, 2004

- (20) **1.**) Consider the initial value problem $ty' + 2y = 4t^2$, $y(1) = 2$.
- (a) What type of first-order differential equation is this?
 - (b) What is the interval on which the solution exists?
 - (c) Find the solution.
- (10) **2.**) Consider the equation $y' + y^2 \sin x = 0$.
- (a) What type of first-order differential equation is this?
 - (b) Find a solution other than $y = 0$.
- (15) **3.**) Consider the equation $t^2y' + 2ty - y^3 = 0$, $t > 0$.
- (a) What type of first order differential equation is this?
 - (b) Find a solution other than $y = 0$. (Hint: Use the substitution $v = y^{-2}$ and first solve the resulting equation for v .)
- (20) **4.**) Consider the equation $y' = \frac{y}{y \sin y - x}$.
- (a) When is an equation of the form $M(x, y) + N(x, y)y' = 0$ exact?
 - (b) Is the given equation exact? Explain using the answer to (a).
 - (c) Solve the given equation (Hint: If it is not exact, use an integrating factor of the form $\mu(y)$ and solve the related exact equation.)
- (20) **5.**) Euler's method can be used to approximate the solution to first-order differential equations of the form $y' = f(t, y)$ at times t_n by a sequence of solution y_n to the first order difference equation

$$y_{n+1} = y_n + f(t_n, y_n)h, \quad y_0 = y(0), \quad n = 0, 1, 2, \dots$$

where $h = t_{n+1} - t_n$ is the step size.

Use Euler's method to find approximate values to the initial value problem

$$y' = 5(t + 1) - 3\sqrt{y}, \quad y(0) = 1$$

at times $t = 1, 2$ using $h = 1$. It is not necessary to simplify your answer.