Mathematics 244: Lab 0

Fall 2004

0. Introduction and Setup.

This lab is intended to introduce you to some of the features of Maple that are useful in solving differential equations and to give you practice preparing a Maple worksheet. Other resources can be found on the web page for Math 244.

Although this lab is for practice only, it is important that you learn how to prepare a Maple worksheet for grading. You should begin by typing your name, and anything else requested by your instructor, on a header line. The projects ask you to interpret the results obtained by Maple, and you should use the **text** feature of Maple to insert your observations into the worksheet (don't write header lines or discussion by hand). Graphs should be generated using the default "inline" option, so that they appear in your worksheet. The **title** option should be used to include a brief description with each graph. The final worksheet should be **edited** to remove any extraneous material.

Earlier versions of these lab descriptions included samples of the Maple instructions used. These snippets of Maple were also available as a "seed file", downloadable from the web page. The ease of obtaining the seed file means that verbatim quotes of Maple instructions in the descriptions can be phased out. The seed file uses an outline structure to mimic the organization of the lab description. This outline should be fully expanded when you print your final copy.

Since the standard behavior of Maple now uses a separate kernel for each worksheet, you can do scratch work on separate worksheets without affecting the worksheet in which you construct your answers. You are encouraged to use this to explore the use of Maple beyond the confines of these projects.

In order to assure that the special commands Maple has for producing plots and solving differential equations are available when they are needed, the worksheet begins `with(plots): with(DEtools):`

1. Expressions, derivatives and graphs.

The seed file contains instructions for producing the Maple expression `f0` representing $e^{\sin x}$ and expressions `f1` and `f2` for its first two derivatives. These three quantities are then graphed on the interval $-2\pi \leq x \leq \pi$ with $-3 \leq y \leq 3$, using red for the function, green for the first derivative, and blue for the second derivative. (Note that the name `fvars` is used for an **expression sequence** that can gives the same **plotting window** to each `plot` instruction, and that the instructions giving names to the separate plots end with colons rather than semicolons to hide the details of the plot structure while the `display` command ends with a semicolon to show the plot. The use of the `title` option is also illustrated here.)

Construct expressions `g0`, `g1` and `g2` for the expression $x\sin(x)\ln(x)$ and its first two derivatives. Then, introduce a variable for the plotting window $-1 \leq x \leq 1$ and $-2 \leq y \leq 2$, and plot these three expressions on the same set of axes in this window, using red for the function, green for the first derivative, and blue for the second derivative.

This graph should **suggest** the behavior of these functions as $x \to 0$. Use `limit(g0,x=0);`, `limit(g1,x=0);`, and `limit(g2,x=0);` to discover what Maple believes to be the limits of these functions.

Next, **modify the plotting window** to: (1) **remove** values of x outside the domain of the function; and (2) **further restrict the domain and range** to a window (of your choice) that better illustrates the behavior as $x \to 0$.

This will require some experimentation. It may be useful to **copy** relevant lines to a new worksheet and experiment with different settings in that worksheet, and then copying your favorite choice of plotting window back to the worksheet that you will submit.

Discussion. Consider the following observations:

(1) Only positive values of x appear in these graphs. What property of the function g causes this?

(2) How do the graphs behave near $x = 0$? What evidence is there in the expressions for $x\sin(x)\ln(x)$ and its first two derivatives to support the conclusions shown by the Maple in the graphs and limits that were calculated?

Replace the *reminder* with a summary of your observations and remove any experiments.)

2. Implicit Functions.

Consider the expression

$$1.5 \ln F - 0.9F + \ln R - 0.8R.$$

Because the expression includes $\ln R$ and $\ln F$, it is only defined for $R > 0$ and $F > 0$. To get an idea of the behavior of the function, it can be graphed. The following instructions introduce a name for the expression and a graph showing contours of the expression.

```
ex2:=1.5*ln(F)-0.9*F+ln(R)-0.8*R;
contourplot(ex2,R=0.2..3,F=0.2..3,title="R versus F");
```

Discussion. The value enclosed by the contours is a local maximum (actually a **global** maximum). Find the location of the maximum (find the derivatives using Maple or by hand calculation, whichever you prefer), and the value of that function there. If one of the variables is fixed, **what happens** to the function as the other variable approaches zero? You may want to experiment with different (R, F) ranges in the `contourplot` instruction, but such experiments should be done a scratch worksheet, and not included in your report. Another useful experiment is to include a **list** of contours in the `contourplot` instruction, as in one of the examples on the help page for this instruction.

3. Simple mechanics.

Suppose a ball of mass m is throw upward from a height h with initial velocity v. If the only force acting is gravity, then Newton's second law of motion says that the mass satisfies the differential equation

$$m\frac{d^2y}{dt^2} = -mg, \qquad (G)$$

where g is the acceleration due to gravity and $y(t)$ is the height of the ball above the ground at time t. Note that the initial conditions on y are $y(0) = h$ and $y'(0) = v$. We now show how Maple can be used to find a formula for $y(t)$. The instructions

```
de3:= diff(y(t),t,t) = -g;
ic3:=y(0)=h,D(y)(0)=v;
ans3:=dsolve({de3,ic3});
eval(de3,ans3);
```

are in the seed file.

The first statement defines the differential equation, giving it the name `de3` (you should recognize (G) even though the common factor m has been removed). Also note that in the use of `diff`, the function y must be referred to as $y(t)$. The second statement defines an initial conditions (as an expression sequence). Note the use of `D` (which stands for derivative) to define the initial condition $y'(0) = v$. The third statement applies the Maple command `dsolve` to a **set** consisting of the equation and initial conditions. The result is an equation giving the value of $y(t)$ that is saved under the name `ans3`. Finally, to verify that the result produced by Maple really is a solution of the differential equation, we use the `eval` command on the equation `de3`.

It remains to check that this solution satisfies the initial conditions. You can apply the `eval` function at $t = 0$ directly to `ans3` to get a result the resembles one of the initial conditions. To verify the initial condition on $y'(t)$, you need to apply `diff` to `ans3`, and then apply `eval`. (The solution of this equation is easy enough that you could probably see that it satisfied all conditions without asking Maple to do these computations.)

Discussion. Why do these computations check that you have a solution to the initial value problem? Give a brief comment to indicate that **each property** has been verified.

4. An example from the textbook.

Exercise 31 in section 1.1 asks you to study the differential equation

$$\frac{dy}{dt} = 2t - 1 - y^2.$$

In particular, you are asked to produce a slope field and use this to determine the relation between an initial condition at $t = 0$ and the behavior as $t \to \infty$. The following instructions graph **numerical solutions** in a modest region of the plane using `DEplot` that suggest the long-term behavior of solutions. They also show how Maple searches for — and finds — a formula for the solution. Finally, they show how Maple can find several terms of a series solution.

```
eq31:=diff(y(t),t)=2*t-1-y(t)^2;
DEplot(eq31,y,t=-1..5,{[y(0)=0],[y(1)=0],[y(3)=0]},
y=-3..3,title="Exercise 31");
infolevel[dsolve]:=3;
dsolve(eq31);
genser:=dsolve(eq31,y(t),'series');
ser1:=dsolve({eq31,y(0)=0},y(t),'series');
Order:=20;
ser2:=dsolve({eq31,y(0)=0},y(t),'series');
```

Discussion:

(1) You gave initial conditions at **three** points — does your graph show three solutions? If not, why not? Describe the likely long-term behavior based on the graph. Then use Maple Help to interpret the explicit solution.

(2) What do the **help pages** tell you about the **exact solution** found by `dsolve`.

(3) Experiment with the **series solution** on a separate sheet. The Maple variable `Order` sets the number of terms of the series to be computed, and `Digits` gives the number of decimal places to keep in numerical computations. Try different values of `Digits`; for each, use the `eval` function at $t = 1.0$, $t = 0.1$, and $t = 0.01$ to compare `ser1` and `ser2` at these points. Summarize your observations by noting the extent to which the two series agree at the given points when `Digits` is large enough.

End of Lab0