Mathematics 244: Lab 2

Numerical Methods

Fall 2005

Section 0: Introduction

Note: this document contains some **cosmetic** changes from Fall 2004 version along with **clarifying statements** about the goals of the project. Work started with the previous version may be completed without consequence in the grade.

In this lab, we shall use Maple's ability to approximate the solution of differential equations by numerical methods and add these solutions to the direction fields that we studied in Lab 1. This allows us to extend our understanding of the solutions of differential equations to equations for which a closed form solution is not available.

Please turn in only the printout of your main Maple worksheet. Include explicit answers to all questions asked, using the **text** feature of Maple to insert them in the worksheet. The final worksheet should be **edited** to remove any extraneous material. Editing may be guided by the **Print Preview** feature available from the File menu or toolbar. In particular, this will identify places where you can use the **Insert** menu to add a **Page Break** to avoid an unsuitable automatic break.

The first step is to obtain the **seed file** from the web page and arrange to save it in your directory on eden. You will be asked to execute the Maple commands that have been placed in this worksheet and to add some of your own. The ease of obtaining the seed file means that verbatim quotes of Maple instructions in the descriptions can be phased out, so not all prepared instructions will be described here.

There is also a **supplementary worksheet** that suggests additional experiments that may lead to better answers to the questions considered in this project.

The file begins with the line `with(plots):  with(DEtools):` in a preliminary section as in previous projects.

Equation 1

The sequence of commands to begin working with Equation 1 have already been entered into the seed file. Execute them to see how Maple can be used to plot the direction field along with the particular solution satisfying this differential equation and the initial condition $y(0) = 0$. The quantities `x1int` and `y1int` are introduced to have standard names for the **ranges** of the variables in **all** graphs in this problem. For the other exercise, corresponding commands can be constructed by copying lines to different places in a worksheet and doing some minor editing.

```
de1:= diff(y(x),x) = 3 + 5*x*y(x)
initval1a:={[y(0)=0]};
x1int:=x=-1..1;y1int:=y=-3..3;
DEplot(de1, y(x), x1int, initval1a,y1int,title="Default view of equation 1");
```

The `DEplot` command has many options which you can read about in the Maple help pages. In particular, it is possible to specify the color of the solution curve by using `linecolor`, the color of the direction field by using `color`, to make the gridwork of lines in the direction field finer by using the `dirgrid` option, and to get a finer resolution in computing the solution with the `stepsize` option. For example, try:

```
opt1:=linecolor = black,color = green, dirgrid= [30,30],stepsize=0.1;
DEplot(de1, y(x), x1int, initval1a,y1int, opt1);
```

By varying the initial conditions, it is possible to approximate special solutions. For example, one type of solution of this equation, e.g. the one with with $y(0) = -1$, is everywhere increasing, whereas another type, e.g. the one with $y(0) = -2$ reaches a maximum value and then becomes rapidly decreasing as $x \to +\infty$. By plotting solutions with other values of $y(0)$ between -2 and -1, you can identify more precisely where this change of behavior occurs. **Produce a graph** with values of $y(0)$ separated by 0.1 containing **exactly one solution** of each type. The **supplementary worksheet** contains instructions to graph the solutions with $y(0)$ taking all the values $-2.0, -1.9, -1.8, -1.7, -1.6, -1.5, -1.4, -1.3, -1.2, -1.1, -1.0$. Use this graph to select the **two** initial values giving solutions with the required property.

Conclude your study of this equation with a **discussion** of the following properties of solutions:

1. In the quadrant where $x > 0$ and $y > 0$, solutions seem to be **steadily increasing**. How can this be proved from the differential equation?
2. Other solutions seem to have a **unique maximum point**. Such points must be **local extrema**, so how can the differential equation be used to identify them? Identify a curve that contains all maximum points of these curves. Note that this curve is **not** a solution; its role is only to identify a useful feature of the solutions that should be shown accurately in the graph of a solution.

Equation 2

Consider the differential equation

$$\frac{dy}{dx} = \left(3x^2 - y^2\right) \sin y$$

with the initial condition $y(0) = -1$.

Introduce the name `de2` for this equation and use the `DEplot` command to plot a direction field and the (numerical) solution of this initial value problem for $-3 \le t \le 6$ and $-4 \le y \le 0$ with **no special options**. The result **will not be satisfactory**, and you should **leave this graph in your worksheet**.

To get a more accurate graph, experiment with different stepsizes. The **supplementary worksheet** contains the choices `opt2:=linecolor=black, color=green, dirgrid=[24,12], stepsize=0.1` and `opt3` with `stepsize=0.05` and the same values of the other options. Copy the commands leading to the `DEplot` command that you just constructed from the main worksheet to the supplementary worksheet, and add one of these options. Try other choices of `stepsize` and select one that gives **what you think is an accurate graph** while not using an excessively small `stepsize`. You may also experiment with changing other options. When you have selected a suitable graph, copy the instructions that you need to produce the graph to the main worksheet. The main worksheet will now contain exactly two graphs based on this equation.

You are now ready to **discuss** what you have learned about this equation.

(1) **Verify** that the constant functions $y = 0$ and $y = -\pi$ are solutions of the equation. Note that you don't need to know anything about how to solve an equation to recognize a solution if it is given to you. You can do this with Maple if you wish, but other methods may be easier. However you do this, include a **text summary** of how you checked this.

(2) How does this show that the solution of the given initial value problem has $0 > y(t) > -\pi$ for all t? (Answer in **text**.)

(3) How do you know that the first plot is **not** correct? What gives you confidence that your second plot accurately shows the shape of the graph of the solution of this equation?

End of Lab 2