METHOD OF UNDETERMINED COEFFICIENTS

Method for finding a particular solution $Y(t)$ of a constant coefficient linear inhomogeneous equation, i.e., one of the form

$$ay''(t) + by'(t) + cy(t) = g(t),$$

for certain special forms of the function g. To use the method you must know the roots of the characteristic equation

$$ar^2 + br + c = 0 \tag{$*$}$$

We use the following notation: $P_n(t)$ and $P_n^*(t)$ denote polynomials in t of degree n; these are always part of the *given* function $g(t)$:

$$P_n(t) = a_0 t^n + \cdots + a_{n-1} t + a_n, \qquad P_n^*(t) = a_0^* t^n + \cdots + a_{n-1}^* t + a_n^*.$$

$Q_n(t)$ and $Q_n^*(t)$ also denote polynomials in t of degree n, but with *undetermined* coefficients; thee are always part of the solution $Y(t)$':

$$Q_n(t) = A_0 t^n + \cdots + A_{n-1} t + A_n, \qquad Q_n^*(t) = A_0^* t^n + \cdots + A_{n-1}^* t + A_n^*.$$

$g(t)$	Form of $Y(t)$	Condition
$e^{\alpha t}$	$Ae^{\alpha t}$ $Ate^{\alpha t}$ $At^2e^{\alpha t}$	if α is not a root of $(*)$ if α is a simple root of $(*)$ if α is a double root of $(*)$
$P_n(t)$	$Q_n(t)$ $tQ_n(t)$ $t^2Q_n(t)$	if 0 is not a root of $(*)$ if 0 is a simple root of $(*)$ if 0 is a double root of $(*)$
$P_n(t)e^{\alpha t}$	$Q_n(t)t^s e^{\alpha t}$	if α is a root of $(*)$ of order s
$ae^{\alpha t}\cos\beta t + be^{\alpha t}\sin\beta t$	$e^{\alpha t}(A\cos\beta t + B\sin\beta t)$	if $\alpha + i\beta$ is not a root of $(*)$
$e^{\alpha t}[P_n(t)\cos\beta t + P_n^*(t)\sin\beta t]$	$e^{\alpha t}[Q_n(t)\cos\beta t + Q_n^*(t)\sin\beta t]$	if $\alpha + i\beta$ is not a root of $(*)$
$e^{\alpha t}[P_n(t)\cos\beta t + P_n^*(t)\sin\beta t]$	$te^{\alpha t}[Q_n(t)\cos\beta t + Q_n^*(t)\sin\beta t]$	if $\alpha + i\beta$ is a simple root of $(*)$

Once you know the form of $Y(t)$, you simply plug it into the original equation and solve for the unknown coefficients.