# Mathematics 244: Lab 0 INTRODUCTION TO MAPLE FOR DIFFERENTIAL EQUATIONS

This lab is intended to introduce you to some of the features of Maple that are useful in solving differential equations and to give you practice preparing a Maple worksheet. Most of the information you need to do this lab is contained in the handout **Instructions for Use of Maple in Mathematics 244**. You can also learn more about the relevant Maple commands by using the **Help** feature of Maple described in that document, paying particular attention to the examples at the end of each **Help** page.

Although this lab is for practice only, it is important that you learn how to prepare your Maple worksheet in the way you will be asked to do it in future assignments on which you will be graded. Be sure to include explicit answers to all questions asked, using the **text** feature of Maple to insert them in the worksheet (don't write them by hand). Insert graphs directly into your worksheet and use the editing capabilities of Maple to remove from the worksheet any extraneous material such as any errors you have made.

In order to use some of the special commands Maple has for producing plots and solving differential equations, you must first type:

```
with(plots):  with(DEtools):
```

1. Find the first and second derivatives of the function $g(x) = \sin(x^2)\ln(x)$. (First define an expression for $g$ and then use Maple's `diff` command.)

2. Using Maple's `plot` and `display` commands plot $g(x)$ and its first two derivatives on the same set of axes over the interval $0.1 \le x \le 2$, using red for the function, green for the first derivative, and blue for the second derivative.

3. The equation

$$y^2/2 + e^y - x^2/2 - e^{-x} = e - 1/2$$

defines $y$ implicitly as a function of $x$. Use the `implicitplot` command to produce a plot of $y$ over the range $-2 \le x \le 4$, $0 \le y \le 4$. Then use the `fsolve` command to find $y(2)$, i.e., set $x = 2$ and solve the resulting equation for $y$.

4. For each value of the constant $c$, the equation

$$y^2/2 + e^y - x^2/2 - e^{-x} = c$$

defines $y$ implicitly as a function of $x$. As $c$ varies, the function $y(x)$ will change, so we get a family of functions corresponding to different values of the constant $c$. Use the `contourplot` command to obtain a plot of a family of solutions over the range $-2 \le x \le 4$, $0 \le y \le 4$.

5. Maple's `solve` command can be used to find exact solutions to some types of equations. For example, try to find the solutions of the equation $x^3 - 7x^2 + 17x - 15 = 0$.

6. For many equations, Maple is not able to give an explicit solution and so uses the notation `RootOf` to describe the solution. For example, note what happens if the `solve` command to used to try to solve the equation in Problem 3 for $y$ (in terms of $x$).

```
u:= solve(y^2/2 + exp(y) -x^2/2 - exp(-x) = exp(1)-1/2,y);
```

To understand Maple's response, note that if you multiply the equation by $2e^x$ and rearrange the terms, then $y$ satisfies

$$2e^{y+x} + y^2e^x - x^2e^x - 2 - 2e^{1+x} + e^x = 0.$$

If the unknown $y$ is replaced by `_Z`, then `_Z` will be a root of the expression displayed by Maple in response to the `solve` command.

7. Suppose a ball of mass $m$ is throw upward from a height $h$ with initial velocity $v$. If the only force acting is gravity, then Newton's second law of motion says that the mass satisfies the differential equation

$$m\frac{d^2y}{dt^2} = -mg,$$

where $g$ is the acceleration due to gravity and $y(t)$ is the height of the ball above the ground at time $t$. Note that $y$ satisfies the initial conditions $y(0) = h$ and $y'(0) = v$. We now show how Maple can be used to find a formula for $y(t)$. Type

```
with(DEtools):
de:= diff(y(t),t$2) = -g;
ivp:={de,y(0)=h,D(y)(0)=v};
dsolve(ivp,y(t));
```

The first statement tells Maple to use one of its special packages for solving differential equations. The second statement defines the differential equation, giving it the name `de` (note the common factor $m$ has been cancelled). Also note that in the use of `diff`, the function $y$ must be referred to as $y(t)$. The third statement defines an initial value problem, consisting of the differential equation and two initial conditions. Note the use of `D` (which stands for derivative) to define the initial condition $y'(0) = v$. The last statement uses the Maple command `dsolve` to solve the initial value problem for the unknown function $y(t)$.

The result of the `dsolve` command is an equation. If we want to verify that the result produced by Maple really is a solution of the initial value problem, we first use the `rhs` command to define the solution as the right hand side of the equation produced by the `dsolve` command. We then use the `diff` and `subs` commands to check that the solution `soln` satisfies the differential equation and the initial conditions. Type in the following to see the result.

```
soln:=rhs(dsolve(ivp,y(t)));
soln1:=diff(soln,t);
soln2:=diff(soln,t$2);
soln2+g;
subs(t=0,soln);
subs(t=0,soln1);
```