Instructions for Use of Maple in Mathematics 244

The computer program **Maple** is a powerful tool which can help you solve a wide range of mathematical problems: it can differentiate, integrate, and otherwise manipulate mathematical formulas, perform arithmetic calculations, plot curves and surfaces in two and three dimensions, solve differential equations, and carry out a variety of other useful mathematical operations. At Rutgers, Maple is available on the student computer **eden**. The instructions in this handout are for a special version of Maple, called **xmaple**, which is customized to run under the X windows system. This means that they apply when you are working on an x-terminal connected to eden. Maple is also available on the Windows PCs and Macintoshes in the public labs. These computers also have a program call "eXceed" that can be used to open an X windows session on eden.

Although the commands you use to do mathematics with Maple will be the same on all these machines, there are differences among computers, such as in the way that files are opened, saved, and printed.

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Getting started: From an x-terminal in one of the public labs (e.g., in ARC) you should log in to **eden**, then start Maple by typing the command `xmaple &` (note that "xmaple" is in lower case; typing the & allows you to continue using your xterm window for other purposes while Maple is running). A Maple window will open. At the top of the Maple window is the *Menu Bar*, consisting of a row of *menu buttons* (**File, Edit** etc.). Underneath the row of menu buttons is the *Tool Bar*, most of which are shortcuts to menu commands. Directly below the *Tool Bar* is the *Context Bar*, which consists of more buttons which are shortcuts to menu commands. If you place the mouse pointer on any of these buttons and hold down the left mouse button, a rough idea of what the button does will appear at the bottom of your Maple window. Finally, below the *Context Bar* is a window with the label *Untitled (1)*. You can now give an instruction to Maple by placing the mouse pointer in

this window (just to the right of the prompt >), typing a Maple command, and then typing a carriage return. Maple carries out the command and prints a response. For example, if you type `1 + 1;` , Maple will respond with **2**. Note that you must type a semicolon after each Maple command, to indicate that you have finished entering the command.

Since you will not be given a Maple manual, you should learn to use Maple by using the built in **Help** facility. To get help on a particular topic, such as the `plot` command, type, after the prompt >, the command `?plot;` . A window will open describing the basic structure of the command and giving examples of its use. To close this window when you have finished with it, click the left mouse button on the rectangle in the upper left corner of the window and then drag the mouse pointer down, releasing the button when **Close** is highlighted. Another way to invoke **Help** is to place the mouse pointer on the word **Help** at the far right of the Menu Bar, click the left mouse button, and drag the mouse pointer down, releasing the button when the desired item (such as **Topic Search**) is highlighted. We shall refer to this process as *choosing* **Topic Search** from the **Help** menu. Fill in the **Topic** box, click the left mouse button on the item and you want, and then click on **OK**.

To continue, you will need to know some basic commands and syntax of Maple.

Arithmetic: The operations addition, subtraction, multiplication, division, and exponentiation are indicated by `+, -, *, /, ^` respectively. All grouping of expressions is done with the left and right parentheses (and). The product xy must be written `x*y`, not `xy` or `x y`; if you type `xy`, Maple assumes you are referring to a variable called "xy". Thus, to enter the expression $(2x + y^2)/(2x + e^x) + 1$ into Maple, you type:

```
(2*x +y^2)/(2*x + exp(x)) + 1;
```

Note that the exponential function is built into Maple and is referred to as `exp`. Similarly, Maple knows the functions `log`, `sin`, `cos`, `tan`, and many more standard functions.

When operating on integers, Maple does exact arithmetic, rather than using decimal approximations. To get a decimal approximation, use the Maple command `evalf`. The Maple command `evalf(4/7,20);` produces a 20 digit approximation to 4/7. Typing only `evalf(4/7);` will produce a 10 digit approximation—as will typing `4.0/7.0;` .

Algebra: To help you do algebraic manipulations, Maple has the commands `expand`, `factor`, and `simplify`, which you can learn about by using the **Help** facility. You can also solve algebraic equations by using the commands `solve` and `fsolve`.

User-defined Functions and Expressions: In Maple, `x^2 - 2*x +3` is an **expression**. You can assign a name to this expression for future use by typing `g:= x^2 - 2*x +3;` . Expressions can contain several variables, as in `h:= y*t - sin(y)`. To evaluate an expression at a particular value, use the Maple command `subs`. For example, `subs(x=2,g);` will produce the value 3. Maple also has a construct called a **function**, with the syntax `f:=x -> x^2 - 2*x +3;` . We will use these only occasionally.

Plots: The basic plotting command in Maple is `plot`. This command has many forms—for example, several functions can be plotted at once—so you should look carefully at the examples given at the end of its **Help** page to get some idea of its flexibility. There are

many other plotting commands in Maple; in Math 251 we used `plot3d` and `implicitplot` and in this course we will use `dfieldplot`, `odeplot`, and `DEplot` (see the section on **Differential Equations** for a description of these commands). Before using the `plot3d` and `implicitplot` commands, as well as some other plotting commands, you must first issue the command `with(plots):` To use the commands for solving and plotting solutions to differential equations, you must first issue the command `with(DEtools):` It is frequently useful to enlarge the size of a plot. To do this, place the mouse pointer in the region occupied by the plot and click the left mouse button. This places a box around the plot and changes the Menu Bar and Control Bar so that new options are offered to manipulate the plot. The plot is resized by placing the mouse pointer at one of the small dots along the edge of the box and dragging the pointer. To enter additional commands, move the mouse down to the next prompt > and click the left mouse button. When viewing three dimensional plots, it is useful to view the plot from different viewpoints. First, place a box around the plot by moving the mouse pointer in the region occupied by the plot and clicking the left mouse button. Now, place the mouse pointer inside the box and while holding down the left mouse button, move the mouse pointer to different positions. A cube will appear which rotates as the mouse pointer moves. When you are at the desired viewpoint, hold down the right mouse button and choose **Redraw**. Explore the effects of using the **Axes** and other commands on the **Menu** Bar. (Note that each time a change is made, you must also use **Redraw**.)

Differentiation and Integration: In addition to performing basic calculations and evaluating standard functions, Maple can also differentiate and integrate functions. The `diff` command differentiates expressions. For example, `diff(x*sin(x), x);` differentiates $x\sin(x)$ with respect to x, and either `diff(x*sin(x), x, x);` or `diff(x*sin(x),x$2);` gives the second derivative. If you have typed `g:=x^2*y;` , then `diff(g,x);` is the first partial derivative of x^2y with respect to x and `diff(g,x,y);` is the mixed second partial derivative.

The command for both definite and indefinite integration is `int`. If Maple cannot evaluate a definite integral exactly, numerical integration may be used. Type `?int` and `?int[numerical]` for details on integration in Maple.

Differential Equations: Maple has a special set of commands to deal with differential equations. To access these, you must first type `with(DEtools):` Maple can obtain exact solutions of some differential equations and can obtain numerical solutions in cases in which it can not find an exact solution. The basic command for this is `dsolve`. There are four types of solutions this command can be used to find: **exact, series, laplace** and **numerical**. To plot the direction field of a single first order differential equation or a system of two autonomous first order equations, the commands `dfieldplot` or `DEplot` may be used. The standard `plot` command is used to graph exact solutions of differential equations found by the `dsolve` command. To plot the numerical solutions found by `dsolve`, the command `odeplot` may be used. Another alternative which directly plots numerical solutions to differential equations is the `DEplot` command. However, this command only gives a graph of the solution. It cannot be used to give the value of a numerical solution at a particular point.

Consider the differential equation $d^2y/dx^2 - y = x^3$. To describe this equation in Maple, we need to communicate that the unknown solution y is a function of x, and to describe the relevant derivatives of y with respect to x. To communicate to Maple that y is the dependent variable and x is the independent variable, we write $y(x)$ instead of y when referring to this variable. Since the first derivative is `diff(y(x),x)` the second `diff(y(x),x$2)`, etc., the differential equation $d^2y/dx^2 - y = x^3$ should be expressed to Maple as `diff(y(x),x$2) -y(x)=x^3` . Notice that Maple echoes it back in symbolic form. To use this equation later we give it a name by typing

```
de:=diff(y(x),x$2) -y(x)=x^3;
```

so that the symbol `de` will refer to this equation. We can use the commands `rhs` and `lhs` to refer to the right or left side of the differential equation. To place other conditions on y(x), we group additional equations together with `de` inside braces. For example the initial value problem

$$\frac{d^2y}{dx^2} - y = x^3, \quad y(0) = 1, \quad y'(0) = 2$$

can be described to Maple via `ivp:={de,y(0)=1,D(y)(0)=2};` . Note the use of D as another notation for differentiation. Any conditions on higher derivatives at a point are described via the D notation as well, for example `D(D(y))(0)=3`.

Once a differential equation has been described to Maple, Maple can attempt to find its general solution, particular solutions, or to plot solutions. We briefly describe the relevant functions here. Find more about a function by typing `?functionname` , for example `?dsolve` will tell you all about the Maple function `dsolve`.

Brief summary of relevant Maple commands for Differential Equations

`dfieldplot` produces a plot of the direction field of a single first order differential equation or a system of two first order autonomous differential equations.

`dsolve(de,y(x));` or `dsolve(ivp,y(x));` In these basic forms, `dsolve` will attempt to solve the differential equation or the initial value problem for $y(x)$ described in `ivp`, returning a formula for $y(x)$ if possible. In the first form, arbitrary constants `_C1`, etc. will be used to express a general solution.

`soln:= dsolve(ivp,y(x),numeric);` In this form `dsolve` finds an approximate solution to the initial value problem described in `ivp`. The output `soln` is a Maple procedure. To get the value of this approximate solution at a particular point, e.g., $x = 2.5$, type `soln(2.5);`. Because the solution is returned as a Maple procedure which produces output of the form `[x= 2.5, y(x) = 6.437]`, the `plot` command cannot be used directly to graph the approximate solution. Instead, use the `odeplot` command. Sometimes it is useful to convert the approximate solution from the form of a Maple procedure into a standard function that just returns the y-value. This can be done by typing `yapprox:= u-> subs(soln(u), y(x));`. A graph of the approximate solution over the interval $0 \leq x \leq 5$ can then be obtained by typing `plot(yapprox, x=0..5);`.

`odeplot` is used to plot numerical solutions of differential equation obtained from using the `numeric` option of `dsolve`.

`DEplot` may be used to plot numerical solutions of differential equations with a set of initial conditions. In the case of a single first order equation or a system of two first order autonomous equations, it can also be used to plot the direction field of the equation, with or without solution curves. In fact, the default option is to also produce the direction field; to suppress the direction field, one must add the option `arrows = NONE`. For computing the numerical solution at a particular point, the `numeric` option of `dsolve` should be used. The default numerical method used by the `DEplot` command is the classical fourth order Runge-Kutta method with a fixed step size. For some problems, the default step size may be too large to produce an accurate solution. This may be corrected by using the `stepsize` option. In general, the default method used by `DEplot` is a poor choice of method, so it is better to use the command with the option `method=rkf45`, which is the default method for the `numeric` option of the `dsolve` command.

Examples of use of Maple commands for Differential Equations

```
with(plots): with(DEtools):
de1:= diff(y(x),x) = - y(x) + 1/(1 + exp(x));
s1:=dsolve(de1,y(x));
t1:=rhs(dsolve({de1,y(0)=-2},y(x)));
plot(t1,x=-1..5);
dfieldplot(de1,y(x), x=-1..5,y= -6..4);
initval:={[y(0)=-2],[y(0)=1]};
DEplot(de1, y(x), x=-1..5, initval,y=-6..4);
t2:=dsolve({de1,y(0)=-2},y(x),numeric);
odeplot(t2,[x,y(x)],-1..5);
```

Linear Algebra: To use Maple's linear algebra commands, you first enter `with(linalg):` To see a list of commands in this package, type `with(linalg);`, i.e., use a semicolon instead of a colon. In this course, we will only use a few of Maple's linear algebra commands: `matadd` to add two matrices, `multiply` to multiply a matrix times a vector, `linsolve` to solve the linear system of equations $Ax = b$, `det` to find the determinant of a matrix, and `eigenvects` to find the eigenvalues and eigenvectors of a matrix.

Maple Worksheets: The window which appears when you start Maple is called a **Maple worksheet.** To complete each computer assignment in Mathematics 244 you are asked to turn in an edited printout of your work; such a printout can be obtained by editing, and then printing, your worksheet. The printout should include only the numerical, symbolic, and graphical output of Maple which is appropriate for the solution of the problems assigned, plus text material in which labels are provided for graphical output and explanations added. Maple includes various editing capabilities which should enable you to produce neat and coherent output, and which we now describe.

To remove an unwanted portion of your Maple worksheet (e.g., a region containing commands that you typed incorrectly or that were not directly relevant to the solution of the exercises), select the region to be deleted by clicking the left mouse button at the beginning, then dragging the mouse across to the end of the portion of the worksheet you

wish to delete. The region should now be highlighted. Then choose **Cut** from the **Edit** menu. To copy a region to a new location, select the region as above, but now choose **Copy** from the **Edit** menu. Then click the left mouse button in the position in which you wish to insert your selected region and choose **Paste** from the **Edit** menu.

To insert text, such as a label for a plot, into your worksheet, click the mouse at the beginning or end of the plot and choose **Text Input** from the **Insert** menu. Now type your label. To continue using your worksheet, move the mouse pointer down to the next prompt > and click the left mouse button. If there is no prompt, insert one by clicking on the prompt symbol > in the Tool Bar Menu.

Sometimes it is useful to be able to place a comment after a Maple command, rather than insert text elsewhere in the worksheet. To do this, enter the sharp symbol #. Everything typed on a line following this symbol will be considered by Maple to be a comment, and therefore not executed.

To make your worksheet less cluttered, it is a good idea to have Maple suppress the output of various commands, e.g., the command `with(plots)` or a command given to assign a name to a plot. To do this, end the command with a colon (:), instead of a semicolon (;).

Printing your Maple Worksheet: To print your Maple worksheet, choose **Print** from the **File** menu or click on the print button in the **Tool Bar** (this is the fourth button from the left). A panel will appear with the options "Output to File" and "Print Command" at the top. Clicking on "Print Command" directs your output to the printer, while clicking on "Output to File" places your output in a file, whose name is given in the adjacent box (the default is *untitled.ps*). If you wish to change this name, click in the box and type in a new name. Since the file will be a postscript file, the file name should have the form *something.ps*. When you have finished making your choices, click on the "Print" button.

Saving your Maple Worksheet: If your work is interrupted, you can save your work so that you can later resume where you left off. To save your Maple worksheet, choose **Save As** from the **File** menu. Then type the name of the file in which you wish to save your worksheet in the **Selection** box where the mouse pointer is. (First delete the asterisk.) The file name should have the form *something.mws*. After you have typed the filename, click on "OK" to save the file. Once you have named the worksheet and saved it in a file, you can save further changes by choosing **Save** from the **File** menu. Maple automatically keeps track of the filename.

Opening a Previously Saved Worksheet: To open a previously saved worksheet, choose **Open** from the **File** Menu. Click on the name of the file you wish to open and then click on **OK**. If you wish to close one of your open worksheets, make it active by clicking in it, and then select **Close** from the **File** menu. Maple will prompt you to save the worksheet if you have changed it since the last save.

Ending your Maple Session: To end your Maple session, choose **Exit** from the **File** menu. A box will open, reminding you that all unsaved work will be discarded. If you have saved what you need or wish to exit anyway, click on "Exit."

Obtaining Copies of the Labs in Worksheet Form: In some of the labs, you will be asked to simply execute a string of Maple commands to learn what they do. To avoid retyping these commands, you can first obtain a modified copy of the lab in worksheet form. This modified copy will omit instructions and problems and contain only strings of Maple commands you are asked to execute. Once you obtain this file, you can access it by following the instructions in the section **Opening a Previously Saved Worksheet:**

The home page of the course contains a section that contains pointers to these Maple worksheets. Follow the instructions there to obtain your personal copy of the worksheet before starting Maple.

Useful Commands and Techniques: If Maple gets hung up in a calculation or is taking too long, click the mouse on the **stop** button in the **Tool Bar** menu (fifth button from the right).

Any Maple command previously entered in your worksheet can be re-executed without retyping it in a new location. Simply move the mouse to the position of the command you wish to execute and hit the **Return** key.

It is often useful to be able to refer later to the result of a computation—the output of some command—in a simple way. To make this possible, simply assign the output of the command to a variable. For example, if you enter `a:= evalf(2*Pi);` then you can later square the result of `evalf(2*Pi);` by typing `a*a;` .

You can assign a name to a plot just as described above for assigning a name to an expression. Several previously named plots can then be displayed on the same graph by using the command `display`. Type `?plots[display]` for details.

If you assign the name `a` as above and then continue your Maple session, you may want to reassign the name `a` to another expression. To do so, first unassign `a` by typing `a:='a';` . To clear all the assigned variables in a Maple session, type `restart;` . One common problem is to try to use such a variable without unassigning it, forgetting that is has already been assigned a value. A simple way to avoid this is to issue the `restart` command before beginning a new problem. This will not change anything on your screen.

Getting Help From Other Students:

The purpose of the Maple assignments is partly to learn about Maple, a very useful program for symbolic, numerical, and graphical computations, and partly to help you understand the material in the course. Just as with other homework assignments, it is permissible and helpful to discuss the Maple labs with other students. However, the Maple labs you are turning in are being graded and will be part of your final course grade. It is therefore expected that the work you hand in is your own in the sense that you have personally input everything that appears on your Maple worksheet, using notation that you have determined, in a style and layout that is your own, and with text comments that reflect the way you finally understand what is being asked. In light of the above, it seems highly unlikely that any two students would turn in nearly identical assignments, and if such an event occurs, the students involved would be considered in violation of the policy on academic integrity and subject to university sanctions.