Mathematics 244: Lab 4 **Systems of First Order Linear Differential Equations**

In this lab we use Maple to find eigenvalues and eigenvectors of matrices, and solve linear systems of algebraic equations and systems of first order linear differential equations.

Please turn in only the printout of your Maple worksheet. Include explicit answers to all questions asked, using the **text** feature of Maple to insert them in the worksheet. Remove from the worksheet any extraneous material and any errors you have made.

Before you start up Maple, first copy the file lab4.mws into your directory from the Web page of the course. After you start up Maple, import this file into Maple by choosing **Open** from the **File** menu as described in the handout **Instructions for Use of Maple in Mathematics 244**. You will be asked to execute the Maple commands that have been placed in this worksheet and to add some of your own.

Note that the command `with(linalg):` must be entered to allow you to use the matrix commands which are part of this special Maple linear algebra package.

1a. Execute the following commands (already entered into Maple worksheet lab4.mws) which show how matrices and vectors are entered into Maple, how Maple adds two matrices, solves the linear system of equations $Ax = b$, multiplies a matrix times a vector, and computes the derminant of a matrix. Note that although the vector `b` is entered as a row vector, Maple considers it to be a column vector.

```
with(plots): with(DEtools): with(linalg):
A:= [[3, 2, 2], [1, 4, 1], [-2, -4, -1]];
matrix(A);
B:= [[-3, -2, -2], [-1, -4, -1], [2, 4, 1]];
matrix(B);
matadd(A,B);
b:= [1,2,3];
x:= linsolve(A,b);
multiply(A,x);
det(A);
```

Note the result of `multiply(A,x);` should give back `b`.

1b. Execute the following commands (already entered into Maple worksheet lab4.mws) which show how Maple calculates the eigenvalues and eigenvectors of a matrix.

```
C:= [[0, 1, 1], [1, 0, 1], [1, 1, 0]];
matrix(C);
eigenvects(C);
```

The output is a sequence which is interpreted as follows: Each entry of the sequence consists of an expression of the form `lam, m, [eigenvector]`, where the first number is the eigenvalue, the second number is the number of times this eigenvalue occurs, and the third entry is a set of linearly independent eigenvectors associated to that eigenvalue. In this case, `C` has the eigenvalue 2, which occurs once with corresponding eigenvector $[1, 1, 1]$ and the eigenvalue -1, which occurs twice with two linearly independent eigenvectors $[-1, 0, 1]$ and $[-1, 1, 0]$. Enter these eigenvectors into your Maple worksheet as `v1`, `v2`, and `v3`, and use Maple to check that $Cv_1 - 2v_1 = [0, 0, 0]$, $Cv_2 + v_2 = [0, 0, 0]$, and $Cv_3 + v_3 = [0, 0, 0]$.

1c. Use Maple to show that any linear combination $v = ev_2 + fv_3$ of v_2 and v_3 is an eigenvector of C corresponding to the eigenvalue -1, for any constants e and f.

2a. Use Maple to find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{pmatrix}.$$

2b. The general solution of the system of differential equations $\mathbf{x}' = A\mathbf{x}$, where A is the matrix of part (a) can be written in the form $\mathbf{x} = \sum_{i=1}^{3} c_i e^{\lambda_i t}\mathbf{w}_i$, where c_i are arbitrary constants and the λ_i and $\mathbf{w}_i$ are the eigenvalues and corresponding eigenvectors of A. If $\mathbf{x}$ also satisfies the initial condition $\mathbf{x}(0) = [1, -2, 4]$, then the c_i are determined by the equation, $[1, -2, 4] = \sum_{i=1}^{3} c_i\mathbf{w}_i$. This equation may be rewritten as a linear system of equations of the form

$$M \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$$

for some matrix M. Determine the matrix M, enter it into your Maple worksheet, and then use `linsolve` to solve the above linear system of equations for c_1, c_2, c_3. Finally, enter into your Maple worksheet the solution of the initial value problem $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = [1, -2, 4]$.

2c. To check your answer, execute the following commands which show how Maple's `dsolve` command can be used to solve this initial value problem directly.

```
dsolve({diff(x1(t),t) = 3*x1(t) + 2*x2(t) + 2*x3(t),
diff(x2(t),t) = x1(t) + 4*x2(t) + x3(t),
diff(x3(t),t) = -2*x1(t) -4*x2(t) - x3(t),x1(0)=1,x2(0)=-2,x3(0)=4},
{x1(t),x2(t),x3(t)});
```