# 244 Differential Equations, Practice Problem 1, Sections 10–12

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**Problem 1: Linear & Separable** Solve the following equations: a) $xy' - 2y = x^4e^x$, $y(1) = 1$.

b) $y' + y^2 \sin x = 0$.

**Problem 2: Exact** Solve the following differential equation:

$$\frac{dy}{dx} = \frac{2\ln y + x}{\sin y - 2x/y}.$$

**Problem 3: Euler** Consider the following differential equation:

$$y' = 5(x+1) - 3\sqrt{y},\ y(0) = 1.$$

Find the approximate values of $y(1)$ and $y(2)$ using the step size $h = 1$.

**Problem 4** Consider the autonomous equation

$$\frac{dy}{dt} = y(y-1)(y-2).$$

Determine the critical points, and classify each of them as asymptotically stable or unstable. Draw the phase line, and sketch several solution curves in the $xy$-plane.

**Problem 5: Dir. field** a) Draw the direction field for $y' = y/x - 1$. Confine your attention to region $x > 0$, but all $y$. Sketch isoclines on which the slope takes values $-1$, $-1/2$, 0, 1. Plot the field along these isoclines. Sketch several solution curves.

b) (extra) Show that $y'' < 0$ for all solutions if $x > 0$.

**Problem 6A** A tank originally contains 100 gal of fresh water. Then water containing $\frac{1}{2}$ lb of salt per galon is poured into the tank at a rate of 2 gal/min, and the mixture is allowed to leave at the same rate. After 10 min the process is stopped, and fresh water is poured into the tank at a rate of 2 gal/min, with the mixture again leaving at the same rate. Find the amount of salt in the tank at the end of an additional 10 minutes.

**Problem 6B: Substitution tricks** a) (Bernoulli) Solve $x^2y' + 2xy - y^3 = 0$. b) Solve the equation $y' + 2xy = 1 + x^2 + y^2$ by substitution $y = x + 1/v$.

**Problem 7: Wronskian, lin. dep., second order** a) (Abel) Find the Wronskian of two solutions of the following differential equation without solving the equation.

$$x^2y'' - x(x+2)y' + (x+2)y = 0.$$

b) Determine whether the following pair of functions is linearly independent or linearly dependent on the following intervals: $(0, \infty)$, $(-\infty, 0)$, $(-\infty, \infty)$.

c) For what values of $k$ will the solutions of $y'' + 2y' + ky = 0$ be oscillations, i.e. of the form $e^{at}\cos\omega t$, $e^{at}\sin\omega t$?