Mathematics 244: Lab 0

Spring 2006

0. Introduction and Setup. This lab is intended to introduce you to some of the features of Maple that are useful in solving differential equations and to give you practice preparing a Maple worksheet. Other resources can be found on the web page for Math 244. These instructions assume that you will prepare a **printed** version of the worksheet to submit for grading.

Although this lab is for practice only, it is important that you learn how to prepare a Maple worksheet for grading. You should begin by typing your name, and anything else requested by your instructor, on a header line. The projects ask you to interpret the results obtained by Maple, and you should use the **text** feature of Maple to insert your observations into the worksheet (don't write header lines or discussion by hand). Graphs should be generated using the default "inline" option, so that they appear in your worksheet. The **title** option should be used to include a brief description with each graph. The final worksheet should be **edited** to remove any extraneous material. Editing may be guided by the **Print Preview** feature available from the File menu or toolbar. In particular, this will identify places where you can use the **Insert** menu to add a **Page Break** to avoid an unsuitable automatic break.

The **Document Mode** introduced in Maple 10 introduces features that we don't use, but may appear spontaneously as you work in Maple. Using the `T` or `[>` tools in the Maple window to insert **text** or **Maple input** lines, respectively, should avoid odd behavior in your worksheets.

Earlier versions of these lab descriptions included samples of the Maple instructions used. These snippets of Maple were also available as a "seed file", downloadable from the web page. The ease of obtaining the seed file means that verbatim quotes of Maple instructions in the descriptions can be phased out. The seed file uses an outline structure to mimic the organization of the lab description. This outline should be fully expanded when you print your final copy.

Since the standard behavior of Maple now uses a separate kernel for each worksheet, you can do scratch work on separate worksheets without affecting the worksheet in which you construct your answers. You are encouraged to use this to explore the use of Maple beyond the confines of these projects. Some suggestions will be in a **supplementary worksheet** for each lab. Be sure to obtain both the seed file and the supplementary worksheet for each project.

In order to assure that the special commands Maple has for producing plots and solving differential equations are available when they are needed, the worksheet begins `with(plots): with(DEtools):`

1. Expressions, derivatives and graphs. The seed file contains instructions for producing the Maple expression `f0` representing $e^{\sin x}$ and expressions `f1` and `f2` for its first two derivatives. These three quantities are then graphed on the interval $-2\pi \le x \le \pi$ with $-3 \le y \le 3$, using red for the function, green for the first derivative, and blue for the second derivative. (Note that the name `fvars` is used for an **expression sequence** that can gives the same **plotting window** to each `plot` instruction, and that the instructions giving names to the separate plots end with colons rather than semicolons to hide the details of the plot structure while the `display` command ends with a semicolon to show the plot. The use of the `title` option is also illustrated here.)

Construct expressions `g0`, `g1` and `g2` for the expression $g(x) = x^{3/2}\sin(x)$ and its first two derivatives. Then, introduce a variable for the plotting window $-2 \le x \le 2$ and $-3 \le y \le 3$, and plot these three expressions on the same set of axes in this window, using red for the function, green for the first derivative, and blue for the second derivative.

This graph should **suggest** the behavior of these functions as $x \to 0$. Use `limit(g0,x=0);`, `limit(g1,x=0);`, and `limit(g2,x=0);` to discover what Maple believes to be the limits of these functions.

Next, **modify the plotting window** to: (1) **remove** values of x outside the domain of the function; and (2) **further restrict the domain and range** to a window (of your choice) that better illustrates the behavior as $x \to 0$. You should **not** attempt to restrict to the **arbitrarily small** interval appearing in the definition of **limit**. Rather, you should aim for an interval that shows the **domain** of $g(x)$ and its derivatives and the **shape** of their graphs, while including $x = 0$ in, or at one edge of, the graphing window.

This will require some experimentation. It may be useful to **copy** relevant lines to the supplementary worksheet and experiment with different settings in that worksheet, and then copying your favorite choice of plotting window back to the worksheet that you will submit.

Discussion. Consider the following observations:

(1) Only positive values of x appear in these graphs. What property of the function g causes this?

(2) How do the graphs behave near $x = 0$? What evidence is there in the expressions for $x^{3/2}\sin(x)$ and its first two derivatives to support the conclusions shown by the Maple in the graphs and limits that were calculated?

2. Implicit Functions.

Consider the expression

$$1.3 \ln F - 0.8F + \ln R - 1.1R.$$

Because the expression includes $\ln R$ and $\ln F$, it is only defined for $R > 0$ and $F > 0$. To get an idea of the behavior of the function, it can be graphed. The following instructions introduce a name for the expression and a graph showing contours of the expression.

```
ex2:=1.3*ln(F)-0.8*F+ln(R)-1.1*R;
contourplot(ex2,R=0.1..4,F=0.1..6,title="R versus F");
```

Note: The graphing windows don't go all the way to $R = 0$ or $F = 0$ because including those values seems to confuse Maple. Limiting the graphs to values that are small, but **reasonable**, gives good plots.

Discussion. The value enclosed by the contours is a local maximum (actually a **global** maximum). (1) Find the location of the maximum (find the derivatives using Maple or by hand calculation, whichever you prefer), and (2) the value of that function there. (3) If one of the variables is fixed, **what happens** to the function as the other variable approaches zero?

You may want to experiment with different (R, F) ranges in the `contourplot` instruction, but such experiments should be done on the supplementary worksheet, and not included in your report. Another useful experiment is to include a **list** of contours in the `contourplot` instruction, as in one of the examples on the help page for this instruction. An example is also present in the supplementary worksheet.

3. Simple mechanics.

Suppose a ball of mass m is thrown upward from a height h with initial velocity v. If the only force acting is gravity, then Newton's second law of motion says that the mass satisfies the differential equation

$$m\frac{d^2y}{dt^2} = -mg, \qquad (G)$$

where g is the acceleration due to gravity and $y(t)$ is the height of the ball above the ground at time t. Note that the initial conditions on y are $y(0) = h$ and $y'(0) = v$. We now show how Maple can be used to find a formula for $y(t)$. The instructions

```
de3:= diff(y(t),t,t) = -g; ic3:=y(0)=h,D(y)(0)=v;
ans3:=dsolve({de3,ic3});
eval(de3,ans3);
```

are in the seed file.

The first statement defines the differential equation, giving it the name `de3` (you should recognize (G) even though the common factor m has been removed). Also note that in the use of `diff`, the function y must be referred to as $y(t)$. The second statement defines an initial conditions (as an expression sequence). Note the use of `D` (which stands for derivative) to define the initial condition $y'(0) = v$. The third statement applies the Maple command `dsolve` to a **set** consisting of the equation and initial conditions. The result is an equation giving the value of $y(t)$ that is saved under the name `ans3`. Finally, to verify that the result produced by Maple really is a solution of the differential equation, we use the `eval` command on the equation `de3`. The result of this command is **an equation** which should **appear to be true**.

It remains to check that this solution satisfies the initial conditions. You can apply the `eval` function at $t = 0$ directly to `ans3` to get a result that resembles **one** of the initial conditions. To verify the initial condition on $y'(t)$, you need to apply `diff` to `ans3`, and then apply `eval`. (The solution of this equation is easy enough that you could probably see that it satisfied all conditions without asking Maple to do these computations, but you should practice the use of Maple when you can verify the results independently, so you will trust its work when you apply **the same method** to check less obvious solutions.)

Discussion. Why do these computations check that you have a solution to the initial value problem? Give a brief comment to indicate that **each property** has been verified.

4. An example from the textbook.

Exercise 31 in section 1.1 asks you to study the differential equation

$$\frac{dy}{dt} = 2t - 1 - y^2.$$

In particular, you are asked to produce a slope field and use this to determine the relation between an initial condition at $t = 0$ and the behavior as $t \to \infty$. The following instructions graph **numerical solutions** in a modest region of the plane using `DEplot`. The portion of these graphs close to the **sides** of the graphing window already suggest the long-term behavior of solutions, without enlarging that window. Other instructions show how Maple searches for — and finds — a formula for the solution.

```
eq31:=diff(y(t),t)=2*t-1-y(t)^2;ic31:={[y(0)=0],[y(1)=0],[y(3)=0]};
DEplot(eq31,y,t=-1..5,ic31,y=-3..3,title="Exercise 31");
infolevel[dsolve]:=3;dsolve(eq31);
```

Discussion:

(1) You gave initial conditions at **three** points — does your graph show three solutions?
(2) If not, why not?
(3) Describe the likely long-term behavior based on the graph.
  (a) Where are the solutions **increasing**? If a solution is increasing at some point, do you expect it to continue to increase, or will it reach a maximum at some larger value of t?
  (b) Are there other initial conditions that lead to solutions that are decreasing for all t? If so, indicate where you would look for such initial values.
(4) Maple found a formula for the solution, but it involves unfamiliar functions. What are those functions, and what does Maple Help say about them?

End of Lab0