The problems on numerical methods in the text, although valuable, don't emphasize the issues of local and global order. Here is a problem (from an old quiz) on that material.

Little Egbert has developed a numerical method for solving ordinary differential equations. He computes the local truncation error in his method and finds that it is of order h^3 when the step size is h. He then his method to compute a solution to a certain problem $y' = f(t, y)$, $y(1) = 3$. He is interested in the value of the solution when $t = 4$, i.e., in $y(4)$.

(a) With step size h, how many steps are required to compute an approximation for $y(4)$?

(b) *Explain why* the overall (global) error in his computation of $y(4)$ might be expected to be order h^2.

(c) Egbert tries his method on a problem for which he knows the exact answer and, using step size $h = 0.05$, finds an error of 0.04000 in his computed approximation to $y(4)$. *About how large* will his error be when he applies the method to the same problem, but with step size 0.025? Assume the method behaves as expected (see (b)).