

NUMERICAL METHODS

Equation:

$$y' = 1 - t + 4y, \quad y(0) = 1;$$

Exact solution:

$$\phi(t) = \frac{1}{16}(4t - 3 + 19e^{4t}).$$

EULER METHOD				
t	$\phi(t)$	Error: $h = 0.1$	Error: $h = 0.01$	Error: $h = 0.001$
0.00	1.0000	0.000000000000	0.000000000000	0.000000000000
0.10	1.6090	0.109041828449	0.013751740108	0.001412901742
0.20	2.5053	0.315329852585	0.040871120233	0.004213922341
0.30	3.8301	0.684138845750	0.091104302592	0.009425890302
0.40	5.7942	1.319826003969	0.180514008294	0.018741559897
1.00	64.8978	30.486312885959	4.860677195539	0.515245173845
2.00	3540.2001	2546.327076436795	510.872232685863	56.039306535802

IMPROVED EULER METHOD (HEUN'S METHOD)				
t	$\phi(t)$	Error: $h = 0.1$	Error: $h = 0.01$	Error: $h = 0.001$
0.00	1.0000	0.000000000000	0.000000000000	0.000000000000
0.10	1.6090	0.014041828449	0.000183376689	0.000001883984
0.20	2.5053	0.041729852585	0.000547103430	0.000005621144
0.30	3.8301	0.093010845750	0.001224210252	0.000012578635
0.40	5.7942	0.184276563969	0.002434950092	0.000025020145
1.00	64.8978	4.959579932511	0.067081377779	0.000689501491
2.00	3540.2001	520.854150633696	7.321248857906	0.075290611317

RUNGE-KUTTA METHOD				
t	$\phi(t)$	Error: $h = 0.1$	Error: $h = 0.01$	Error: $h = 0.001$
0.00	1.0000	0.000000000000	0.000000000000	0.000000000000
0.10	1.6090	0.000108495116	0.000000014622	0.000000000002
0.20	2.5053	0.000323701474	0.000000043626	0.000000000004
0.30	3.8301	0.000724336599	0.000000097624	0.000000000010
0.40	5.7942	0.001440733519	0.000000194184	0.000000000020
1.00	64.8978	0.039696355450	0.000005351317	0.000000000551
2.00	3540.2001	4.333368151289	0.000584343975	0.000000060213