Forced Oscillations and Beats
Amplitude of forced oscillations

The figure on the next slide shows the amplitude of forced oscillations as a function of the dimensionless forcing frequency $\omega/\omega_0$, where $\omega$ is the actual forcing frequency and $\omega_0$ the natural frequency of the oscillator. Plots are given for six different values of the damping frequency, expressed in terms of the dimensionless coupling constant $\Gamma$.

Observations: Very small damping leads to a peak in the amplitude when the driving frequency is approximately equal to the natural frequency. As the damping increases, the peak becomes smaller and shifts toward lower frequencies. For $\Gamma \geq 2$ there is no peak; amplitude decreases steadily with frequency.
\[ \Gamma = 0.05, 0.2, 0.6, 1.0, 2.0, 4.0 \]
Beats

The next two slides show the oscillations of a driven oscillator which is undamped. (The slides differ only in the scale used on the time axis.) The natural frequency $\omega_0 = 20$ and the driving frequency $\omega = 21$ are nearly equal, and through the formula

$$\cos \omega t - \cos \omega_0 t = 2 \sin \left( \frac{\omega - \omega_0}{2} t \right) \sin \left( \frac{\omega + \omega_0}{2} t \right)$$

this leads to beats: oscillations at a frequency close to the driven and natural frequencies which are modulated by a low frequency amplitude.
\[ \omega = 21, \quad \omega_0 = 20 \]
$\omega = 21, \quad \omega_0 = 20$