The Predator-Prey System

(Lotka-Volterra equations)

If $x$ is the prey species and $y$ the predator, then the Lotka-Volterra equations are

$$x' = x(a - \alpha y), \quad y' = y(-c + \gamma x)$$

where $a, c, \alpha,$ and $\gamma$ are positive constants. For the graphs that follow we take $a = 1.4$, $c = 2.0$, $\alpha = 0.8$, and $\gamma = 1.2$.

We plot the phase plane, followed by solution curves for two different initial conditions. It is a good exercise to trace a solution around a trajectory in the phase plane, noticing when $x$ and $y$ increase and decrease, and to compare the result with the solution curves. In each case you should see that the predator lags behind the prey by about 1/4 of a period.
Slope field, nullclines, and trajectories: predator-prey model
Solution curves (prey is red, predator blue) for $x(0) = y(0) = 1$
Solution curves (prey is red, predator blue) for $x(0) = y(0) = 0.4$