

(a) $\mathbf{w}_1 \cdot \mathbf{w}_2 = 0$ so they form an orthogonal set. The vectors $\mathbf{n}_1 = \mathbf{w}_1/|\mathbf{w}_1| = [1/3, 1/3, 1/3]$, $\mathbf{n}_2 = \mathbf{w}_2/|\mathbf{w}_2| = [1/2, 0, -1/2]$, are an orthonormal basis, so the projection is $\mathbf{w} = (\mathbf{v} \cdot \mathbf{n}_1) \mathbf{n}_1 + (\mathbf{v} \cdot \mathbf{n}_2) \mathbf{n}_2 = [2, 2, 2] = [3/2, 2, 5/2]$,

(b) The Gram-Schmidt process gives:

$$\mathbf{v}_1 = \mathbf{u}_1 = [1, 1, 0, 0]$$

$$\mathbf{v}_2 = \mathbf{u}_2 - (\mathbf{u}_2 \cdot \mathbf{v}_1)/(\mathbf{v}_1 \cdot \mathbf{v}_1)\mathbf{v}_1 = [-1/2, 1/2, 1, 0]$$

$$\mathbf{v}_3 = \mathbf{u}_3 - (\mathbf{u}_3 \cdot \mathbf{v}_1)/(\mathbf{v}_1 \cdot \mathbf{v}_1)\mathbf{v}_1 - (\mathbf{u}_3 \cdot \mathbf{v}_2)/(\mathbf{v}_2 \cdot \mathbf{v}_2)\mathbf{v}_2 = [-1/3, 1/3, -1/3, 1]$$

Note that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are orthogonal.