Math 250-C2 Extra Review Questions for Final Exam Fall 2002

Note: Questions similar to those on the 11 quizzes and the two midterm exams will appear on the final exam. You should be sure to review your work on these tests. The following are *additional* questions of the type that will appear on the final exam. (This list of questions is *not* a sample final exam.)

True/False Questions: Classify the following statements as TRUE or FALSE. Justify each answer TRUE with a brief argument and each answer FALSE with a counterexample. When your answer is FALSE, change the statement in some (small) way so that it becomes true.

1. There is a 2×2 symmetric matrix A with eigenvalues $1, 2$ and eigenvectors $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

2. If $\mathbf{v}$ is an eigenvector for the 4×4 matrix A with eigenvalue 3, then $\mathbf{v}$ is an eigenvector for the matrix A^2 with eigenvalue 3.

3. If P is an 3×3 matrix whose columns are eigenvectors of an 3×3 symmetric matrix, then P is an orthogonal matrix.

4. Suppose $A = [\mathbf{v}_1 \mid \mathbf{v}_2 \mid \mathbf{v}_3]$ is a 3×3 matrix with columns $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ and B is the matrix $[\mathbf{v}_1 - 5\mathbf{v}_2 \mid 7\mathbf{v}_3 \mid \mathbf{v}_2]$. Then $\det B = -35 \det A$.

5. There are vectors $\mathbf{u}, \mathbf{v} \in \mathbf{R}^3$ with $||\mathbf{u}|| = 1, ||\mathbf{v}|| = 2$ and $||\mathbf{u} + \mathbf{v}|| = 4$.

Longer Problems: Answer the following questions in detail.

6. Use the least squares method to find the best choice of a line $y = a_0 + a_1 x$ to fit the (x, y) data points $(-1, 1)$, $(0, 0)$, $(1, 3)$, $(2, 4)$. Plot the line and the data points in the (x, y) plane.

7. Let W be the subspace of $\mathbf{R}^3$ defined by the equation $2x_1 - 3x_2 - 4x_3 = 0$.

(a) Find an orthonormal basis for W.

(b) Find an orthonormal basis for $W^\perp$.

(c) Decompose the vector $\mathbf{v} = [1, 1, 1]$ as $\mathbf{v} = \mathbf{w} + \mathbf{z}$, where $\mathbf{w} \in W$ and $\mathbf{z} \in W^\perp$.

(d) Find the 3×3 matrix P that projects $\mathbf{R}^3$ onto W.

8.(a) Find the eigenvalues and eigenvectors of the symmetric matrix $\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$.

(b) Verify that the eigenvectors are mutually orthogonal. Then find an orthogonal matrix P so that $P^{\mathrm{T}} A P$ is diagonal.

(c) Give the spectral decomposition of A as a linear combination of projection matrices.

(d) Consider the quadratic form $q(x, y) = 2x^2 + 4xy - y^2$. Use the results from **(a)** and **(b)** to find a change of variables

$$\begin{aligned} x &= as + bt \\ y &= cs + dt \end{aligned}$$

and numbers α, β so that $q(x, y) = \alpha s^2 + \beta t^2$