Apr. 19, 2001 QUIZ, MATH 250–C3 Name: ______________________

Signature: ______________________

Determine which of the following matrices are diagonalizable. Briefly explain your reasoning in each case. It is not required to diagonalize the matrices.

1. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 10 \end{bmatrix}$

Solution: Diagonalizable. Reason: the matrix is triangular, so the eigenvalues are the diagonal entries. The four eigenvalues are in particular all different, so the matrix is diagagonalizable.

2. $\begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$

Solution: Not diagonalizable. The characteristic equation is $\det \begin{bmatrix} 3-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix} = 0$, or $(\lambda-2)^2 = 0$. But $A - 2I = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ has rank 1, so only one LI eigenvector can be found. Since two cannot be found, the matrix is not diagonalizable.

3. $\begin{bmatrix} 0 & 1 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Solution: Diagonalizable. The characteristic equation is $\det \begin{bmatrix} -\lambda & 1 & 0 \\ 4 & -\lambda & 0 \\ 0 & 0 & 2-\lambda \end{bmatrix} = 0$, or $(2-\lambda)(\lambda^2 - 4) = 0$ (expand on the last row), or $(2-\lambda)^2(2+\lambda) = 0$. Diagonalizability therefore hinges on whether the geometric multiplicity of $\lambda = 2$ (the only repeated eigenvalue) comes up to its algebraic multiplicity, which is 2. Now $A - 2I = \begin{bmatrix} -2 & 1 & 0 \\ 4 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ has rank 1, so two LI eigenvectors can be found corresponding to $\lambda = 2$ and the geometric multiplicity of $\lambda = 2$ is 2.

4. $\begin{bmatrix} 3 & 7 & 7 \\ 7 & 13 & 7 \\ 7 & 7 & 23 \end{bmatrix}$

Solution: Real symmetric matrices are diagonalizable.