

```

> with(plots):
Warning, the name changecoords has been redefined
> r:=[u/v,u*v];

```

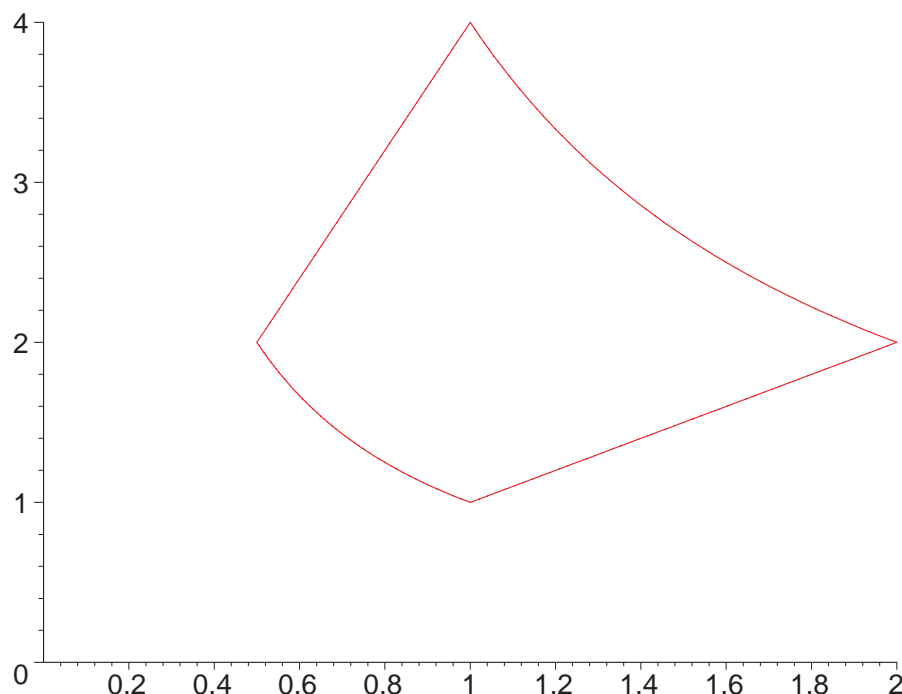
$$r := \left[\frac{u}{v}, u v \right]$$

Here, r is used as the general name for [x,y] -- a vector to a point in physical space. The functions here are the new coordinates that we introduce to compute properties of the regions sketched below.

```

> side1:=plot([subs(u=1,r[1]),subs(u=1,r[2]),v=2..1]):
> side2:=plot([subs(v=1,r[1]),subs(v=1,r[2]),u=1..2]):
> side3:=plot([subs(u=2,r[1]),subs(u=2,r[2]),v=1..2]):
> side4:=plot([subs(v=2,r[1]),subs(v=2,r[2]),u=2..1]):
> display({side1,side2,side3,side4},view=[0..2,0..4]);

```



The change of coordinates is one to one between the first quadrants in (u,v) space and (x,y) space. Each (u,v) determines a unique (x,y) because it is given as a function. If both u and v are positive, so are the computed values. Conversely, if we try to solve these equations for u and v, we are led first to $u^2=xy$, so we need to take a square root. Insisting on a positive square root fixes u and then $v=u/x$ (for example). Thus value of these coordinates when describing this figure is that the boundary arcs are grid curves with respect to u and v (as can be seen from the formulas used to draw them).

We want to find area and first moments of this region in order to locate the centroid. These are the integrals of 1, x, and y over the region. Green's theorem tells us that the result of integrating first with respect to x gives the line integrals of $x dy$, $x^2/2 dy$, and $xy dy$, respectively counterclockwise around the boundary. We get the integrals over these four arcs for the three integrals below using what we hope is an obvious notation.

```

> x1:=subs(u=1,r[1]);x2:=subs(v=1,r[1]);x3:=subs(u=2,r[1]);x4:=subs(

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v=2,r[1]);
> xx1by2:=subs(u=1,r[1]*r[1]/2);xx2by2:=subs(v=1,r[1]*r[1]/2);
> xx3by2:=subs(u=2,r[1]*r[1]/2);xx4by2:=subs(v=2,r[1]*r[1]/2);
> xy1:=subs(u=1,r[1]*r[2]);xy2:=subs(v=1,r[1]*r[2]);xy3:=subs(u=2,r[
1]*r[2]);xy4:=subs(v=2,r[1]*r[2]);


$$x1 := \frac{1}{v}$$


$$x2 := u$$


$$x3 := 2 \frac{1}{v}$$


$$x4 := \frac{1}{2} u$$


$$xx1by2 := \frac{1}{2} \frac{1}{v^2}$$


$$xx2by2 := \frac{1}{2} u^2$$


$$xx3by2 := 2 \frac{1}{v^2}$$


$$xx4by2 := \frac{1}{8} u^2$$


$$xy1 := 1$$


$$xy2 := u^2$$


$$xy3 := 4$$


$$xy4 := u^2$$

> dyu:=diff(r[2],u);dyv:=diff(r[2],v);

$$dyu := v$$


$$dyv := u$$

> A1:=int(subs(u=1,x1*dyv),v=2..1);A2:=int(subs(v=1,x2*dyu),u=1..2);
A3:=int(subs(u=2,x3*dyv),v=1..2);A4:=int(subs(v=2,x4*dyu),u=2..1);

$$A1 := -\ln(2)$$


$$A2 := \frac{3}{2}$$


$$A3 := 4 \ln(2)$$


$$A4 := \frac{-3}{2}$$

> Ax1:=int(subs(u=1,xx1by2*dyv),v=2..1);Ax2:=int(subs(v=1,xx2by2*dyu
),u=1..2);Ax3:=int(subs(u=2,xx3by2*dyv),v=1..2);Ax4:=int(subs(v=2,
xx4by2*dyu),u=2..1);

```

$$Ax1 := \frac{-1}{4}$$

$$Ax2 := \frac{7}{6}$$

$$Ax3 := 2$$

$$Ax4 := \frac{-7}{12}$$

```
> Ay1:=int(subs(u=1,xy1*dyv),v=2..1);Ay2:=int(subs(v=1,xy2*dyu),u=1.2);Ay3:=int(subs(u=2,xy3*dyv),v=1..2);Ay4:=int(subs(v=2,xy4*dyu),u=2..1);
```

$$Ay1 := -1$$

$$Ay2 := \frac{7}{3}$$

$$Ay3 := 8$$

$$Ay4 := \frac{-14}{3}$$

```
> A:=A1+A2+A3+A4;Ax:=Ax1+Ax2+Ax3+Ax4;Ay:=Ay1+Ay2+Ay3+Ay4;
```

$$A := 3 \ln(2)$$

$$Ax := \frac{7}{3}$$

$$Ay := \frac{14}{3}$$

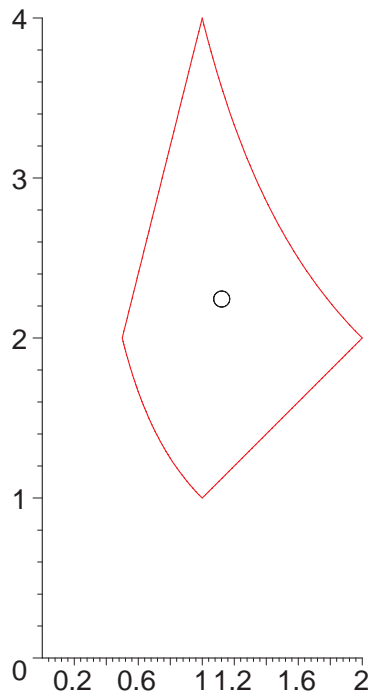
```
> xbar:=evalf(Ax/A);ybar:=evalf(Ay/A);
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$$xbar := 1.122096143$$

$$ybar := 2.244192287$$

```
> centroid:=plot([xbar+0.025*cos(t),ybar+0.025*sin(t),t=-Pi..Pi],color=BLACK):
```

```
> display({side1,side2,side3,side4,centroid},view=[0..2,0..4],scaling=CONSTRAINED);
```



This shows a CONSTRAINED view of the region with a small circle around the centroid.

These integrals can be found directly as double integrals in the uv plane. For this we need the Jacobian

```
> J:=diff(r[1],u)*diff(r[2],v)-diff(r[2],u)*diff(r[1],v);
```

$$J := 2 \frac{u}{v}$$

```
> Auv:=int(int(J,u=1..2),v=1..2);Axuv:=int(int(r[1]*J,u=1..2),v=1..2);
Ayuv:=int(int(r[2]*J,u=1..2),v=1..2);
```

$$Auv := 3 \ln(2)$$

$$Axuv := \frac{7}{3}$$

$$Ayuv := \frac{14}{3}$$

The same results as before. You can supply details. Here are some steps:

```
> J*r[1];
```

$$2 \frac{u^2}{v^2}$$

```
> J*r[2];
```

$$2 u^2$$

Enjoy!