Examples of Surfaces. This section describes a few simple examples of surfaces. Generally, surfaces will be given by an equation $g(x, y, z) = 0$.

Cylinders. An equation in which one of the variables does not appear describes a **cylinder**. All values of the missing variable are allowed while the included variables describe a curve in one of the coordinate planes that gives the common cross-section. Note that the shape of the cross-section is arbitrary.

Cones. Another way to trace a surface with lines is to take a curve in the plane $z = 1$ and join all points on the curve to the origin. Then a point (x, y, z) with $z \neq 0$ lies on this surface if and only if $(x/z, y/z, 1)$ lies on the given curve. If the curve has a polynomial equation, it can be multiplied by a power of z to obtain a polynomial equation of the cone. This equation has the special property that all terms have the same degree.

Conversely, all such **homogeneous** equations define cones.

Quadrics. A surface with an equation of second degree is called a **quadric**. Several such surfaces are explored in Maple Lab 2. Only the special cases with equations

$$Ax^2 + By^2 + Cz^2 + J = 0$$

or

$$Ax^2 + By^2 + z = 0$$

will be considered here since the general equation can be put in one of these forms by a rotation and translation of coordinates.

A major tool in recognizing these surfaces is to consider the intersections with the coordinate planes.

Figures of Rotation. If $A = B$, the sections in the xz plane and the yz plane look the same. Indeed, one has the same section in any plane through the z axis. If we denote the distance from the z axis by r, then $r^2 = x^2 + z^2$ and the equation depends only on r and z. This r is the r of **polar coordinates** in the xy plane. More generally, we can use polar coordinates r and θ as an alternative to the rectangular coordinates x and

y, keeping a z coordinate with the same interpretation. This gives the **cylindrical coordinates** in space that will be considered later in this course.

Changes of Scale. If x is replaced by cx for some constant c without changing the other coordinates, the effect may be described by changing the scale on the x axis without changing the graph. If you prefer to have fixed scales on the axes, this corresponds to shrinking or stretching uniformly in a direction parallel to the x axis. Such transformations take circles into ellipses.

Graphs of Functions. One of the easiest descriptions of surfaces is $z = f(x, y)$. The **implicit function theorem** says that our general formula $g(x, y, z) = 0$ may be assumed to be of this special form in a neighborhood of any point with $g_3(x, y, z) \neq 0$. Here g_3 refers to the partial derivative with respect to the third variable.

Some examples.

#9. $x^2 - y^2 + z^2 = 1$.

#11. $4x^2 + 9y^2 + 26z^2 = 36$.

#15. $y^2 = x^2 + z^2$.

#31. $z = x^2 + y^2 + 1$.