

This lab contains ten problems intended to introduce you to some of the basic features of Maple and to give you practice preparing a Maple worksheet. Most of the Maple commands you need are in the “seed file,” but there are several places where you are asked to interpret results. Additional information you need to use Maple is in the .html/.pdf documents **Instructions for Use of Maple in Mathematics 251**.

This lab is intended for practice only: although the grade will be ignored in computing your grade for the course, the lab will be graded using the same standards that will be applied to later labs. Use this lab to learn how to prepare Maple worksheets, so that you will be ready to do future assignments on which you will be graded. In particular, when you write up this lab be sure to include explicit answers to all questions asked, using the **text** feature of Maple to insert them in the worksheet. Also use the **text** feature of Maple to include your name and section number at the top of the worksheet (do NOT write any of this material in by hand). Use the editing capabilities of Maple to remove from the worksheet any extraneous material such as any errors you have made—or any cries for help.

It is said that the first thing people do when they are introduced to Maple is to compute many-digit approximations to  $\pi$ . To get 100 decimal digits, one needs only to enter `evalf(Pi,100);`. However, the last thing on the agenda of this practice lab will be to investigate ways in which one (or Maple) might do this without relying on built-in functions, using Maple only as a “big calculator that remembers trig identities.”

### Problems

1. The first thing we’ll do is to use Maple’s `simplify` command to simplify the expression

$$\frac{9x^2y - 4y^3}{3x^3y + x^2y^2 - 2xy^3}.$$

Only the beginning of this expression is in the seed file, so you will be able to feel what it’s like to type in a Maple expression. This one will begin as

```
(9*(x^2)*y - 4*y^3)/ ...
```

Notice that multiplication is always represented by `*`—“juxtaposition” will not work—and that parentheses will be needed around the numerator (and the denominator) of the fraction. We can also use “unnecessary” parentheses, like the ones around `(x^2)`, whenever it seems there might be some ambiguity. Furthermore, it will be good to introduce a variable named `h` to represent the expression—then one won’t have to retype it every time one needs it again. Thus the line of Maple you will type will begin like

```
h := (9*(x^2)*y - 4*y^3)/ ...
```

and don’t forget to end the line with a semicolon (`;`) before you hit **enter**. The “colon-equals” relation `:=` tells Maple that this line gave an **assignment** (like the equal-sign in many computer-programming languages) rather than asking for a comparison or solving an equation: now when Maple sees `h` it will “think” of the fully-expanded expression that you typed. {Note: if the cursor falls down into the next problem after you hit **enter**, then (1) use the mouse to put it at the end of this problem; (2) click on the “[>” icon on the toolbar at the top of the screen to restore the Maple command cursor.}

Just eyeballing this expression reveals a common factor in the numerator and denominator: we want to use Maple’s ability to do algebra to remove common factors, even non-obvious ones. Because we are working with **expressions** and not **functions**, Maple will distinguish between `h` and its simplified form; therefore, we can save repeating work already done by saving the simplified form of `h` under a new name, let’s say `k` (though you may use whatever letter you wish). So first look at the simplified form by typing `simplify(h);`, and then take another look at `h`. Has it changed? (Put the answer in your writeup, using **text**). Next, repeat the `simplify` command, but now assign the result to a new variable by typing (say) `k := simplify(h);`. Both variables will be used in the next problem.

2. Maple has a command called **subs** that will plug in numbers (or other expressions) wherever variables occur in an expression. To do this problem, begin by entering `?subs`; for the Maple help file that gives the syntax (= correct form) for using **subs**, as well as **helpful examples** of its use that you can take as models. Now use this command to evaluate the two expressions (turned into variables) called **h** and **k** of Problem 1 above, at the point  $x = 2$ ,  $y = 3$ . In your writeup of this lab, explain the difference (using **text**).

3. In this example, you use Maple's **plot** command to try to see whether the function  $x^x$  has any local maxima or minima on the interval  $0 \leq x \leq 4$ . Try the instruction `plot(x^x,x=0..4);`. You will see that it gives a graph that shows some features of the function on the given interval, but is too "coarse" to give a clear idea of local maxima or minima. To get a more useful graph, you can also restrict the second coordinate. Try the instruction `plot(x^x,x=0..2,y=0..4);`, which will limit the range shown and also introduce the label  $y$  for the vertical axis. Then choose ranges for both variables to give one more graph that illustrates a smaller region that is more useful for studying local maxima or minima. {Note:  $x^x$  is not defined for  $x = 0$ , but except for Maple V, release 5, this has caused no difficulty in the past. If it gives you trouble, try "`x=0.1..4`" instead of "`x=0..4`," etc.} The true minimum can be found by calculus.

4. In this example we see whether Maple can do calculus. Find the first, second, and third derivatives of the expression  $f = e^{x^2}$ . (First define an expression for  $f$ . You may get better results using the function "`exp(x^2)`" than using "e to the x to the 2," but if you use the latter, notice that "`e^x^2`" is ambiguous unless you insert parentheses. [And if you don't remember why these functions are the same, re-read your freshman calculus book!] Next, use Maple's **diff** command repeatedly to introduce new named expressions for the higher derivatives—avoid using previously defined variables like **h**. Typing `?diff;` will show you how to use **diff**.) Form a **list** of these expressions—remember "`?list;`"—and use the **subs** command to evaluate these derivatives at  $x = 0$ . Some of these terms are zero (if they look more complicated than zero, try applying **simplify** or **evalf** to them). Do you think this is the beginning of a pattern? (Give a brief **text** answer in your finished writeup, indicating other things that might be done to test for a pattern.)

5. In this example, we try to find a numerical (approximate) solution to a mixed algebraic-trigonometric equation. Begin by using `plot({x,cos(x)},x=0..1);` to graph  $y = x$  and  $y = \cos(x)$  on the same set of axes. The graphs intersect at a point where  $x = y = \cos(x)$ . Use Maple's **fsolve** command to find the  $x$ -coordinate of this point (remember, if you need help in using Maple commands, start with help requests like `?plot;` or `?fsolve;`). Check by finding the cosine of the value of  $x$  that **fsolve** produced. Describe the results in your **text** writeup.

6. In this example, we see if Maple can do integral calculus. First find the syntax of Maple's **int** command with `?int;`, and then use the command to find the indefinite integral  $\int x^2 \cos(x) dx$ . Finally, find the definite integral  $\int_0^\pi x^2 \cos(x) dx$ . {Note that Maple's idea of  $\pi$  is called **Pi**—capitalization is important!}

7. Maple's **plot** command can also be used to plot parametric equations. For example, the command `plot([2*sin(t),2*cos(t),t=-Pi..Pi]);` plots a circle of radius 2; enter this command to see the circle (an ellipse on most monitors!). Then use this command to plot (in your worksheet) the equations

$$x(t) = \cos(t)(1 - 2 \sin(3t)), \quad y(t) = \sin(t)(1 - 2 \sin(3t)) \quad \text{for } 0 \leq t \leq 2\pi.$$

8. To obtain special plots, it is first necessary to issue the Maple command `with(plots):`. Do this, noticing that ending with a colon ":" rather than the usual semicolon ";" keeps a large list of irrelevancies off your screen {if you don't believe this, try "`with(plots);`"}. The instructors have chosen to use this example to graph an exact solution of the system of differential equations

$$\begin{aligned} \frac{dR}{dt} &= 2R - 1.2RF \\ \frac{dF}{dt} &= -F + 0.9RF \end{aligned}$$

with the initial conditions  $R = F = 1$ ; systems of this form are important in biomathematics. Because Math 251 is not a differential-equations course, we have to furnish you with these functions, which form the arguments of the next Maple command you should execute:

```
implicitplot(2*ln(F) -1.2*F=-ln(R)+0.9*R -2.1,F=0..3,R=0..3); .
```

Now (by eyeball) estimate the largest values of the variables  $R$  and  $F$  on this curve, and put them in your writeup (in **text**).

9. Use the Maple command `plot3d` to obtain a plot (in your worksheet) of

$$z = y(1 - 10xy)e^{-x^2-y^2}$$

over the range  $-3 \leq x \leq 3$ ,  $-3 \leq y \leq 3$ . (This requires the **plots** package, but Maple remembers that you loaded it in the previous problem.) Rotate the plot until you get a good view of the surface. Then use the **Context Bar** (or menu) to change the **Axes** option to **Boxed** to get a better idea of the function values. Rotate the plot again until you are able to estimate (to about one decimal place) the maximum value of the function. Enter this value into your Maple worksheet (with a sentence stating what it is) using the **text** feature of Maple.

10. Well, with this much experience we can tackle the problem of approximating  $\pi$  by something more than button-pushing. Computations like the one we are about to do are based on formulas that show that numbers like  $\pi$  differ by an amount known to be small from something that can be computed exactly. For example, the Maclaurin series for  $\arctan(x)$  converges sufficiently quickly that about 70 terms suffice to find  $\arctan(1/5)$  to 100 decimal places. In 1706, this observation led J. Machin to compute  $\pi$  to 100 decimal places using the formula

$$\pi/4 = 4 \arctan(1/5) - \arctan(1/239).$$

You can retrace his steps without heavy hand computation, using the capabilities of Maple.

Why would anyone think this formula would be true? The reason is that a certain multiple of  $\arctan(1/5)$  is very close to  $\pi/4$ . To see this, one needs to do some algebraic manipulation of the tangent function. The command `expand(tan(A+B))`; will give the addition formula for the tangent function. You can then use new variables, for example, `a` and `b`, in place of the tangents of  $A$  and  $B$  respectively to define an expression in two variables, say `m := ???`; which will give the tangent of  $A+B$  if one sets  $a = \tan(A)$  and  $b = \tan(B)$ . Do this, beginning with  $A = B = \arctan(1/5)$ , so that  $a = b = 1/5$ ; then you know the value of  $\tan(2 \arctan(1/5))$ , which is a rational number (use of `simplify` with `subs(...)` as its argument may be necessary to get the number to look “nice”). These numbers are exact, because Maple gives them to us as fractions without using decimal approximations as a pocket calculator would do. Among the numbers  $1/5$ ,  $\tan(2 \arctan(1/5))$ ,  $\tan(3 \arctan(1/5))$ , ... will soon appear a number that is very close to (but slightly larger than) 1. This number is the tangent of an angle—call the angle  $A$ , but we can identify it as a certain small whole-number multiple of  $\arctan(1/5)$ —that is very close to  $\pi/4$  (why?). If we define another angle  $B$  by  $B = A - \pi/4$ , then since we know  $\tan(\pi/4) = 1$  and we know  $\tan A$ , the addition formula for the tangent will allow Maple to find the exact value of  $\tan(B)$ , again as a fraction. Have Maple do this, and you will see where Machin got his formula: because you will now know that  $\pi/4 = A - B =$  a certain multiple of  $\arctan(1/5) - B$ , and since we know  $\tan B$  we can express  $B$  as the arctangent of ... guess what?

The Maple commands `piOver4:=4*arctan(1/5) - arctan(1/239)`; (which shows that Maple will accept unusual names for variables) and `evalf(4*piOver4,100)`; will now show you where the numbers you got at the beginning of this Maple lab came from. The way Machin used his formula was to use the Maclaurin series for the two numbers  $\arctan(1/5)$  and  $\arctan(1/239)$  to get very good approximations for these two arctangents, and then to plug those approximations into his formula. The fact that these Maclaurin series are alternating series also helped him. Interested students might try approximating these arctangents using a few terms of the Maclaurin series and seeing how good an approximation to  $\pi$  (or  $\pi/4$ ) they get.