

Mathematics 251: Lab 0 INTRODUCTION TO MAPLE

This lab contains ten problems intended to introduce you to some of the basic features of Maple and to give you practice preparing a Maple worksheet. Most of the Maple instructions are in the “seed file”, but there are several places where you are asked to interpret results. Additional information you need to do use Maple is contained in the handout **Instructions for Use of Maple in Mathematics 251**.

This lab is for practice only: although the grade will be ignored in computing your grade for the course, the lab will be graded using the same standards that will be applied to later labs. Use this lab to learn how to prepare your Maple worksheet in the way you will be asked to do it in future assignments on which you will be graded. In particular, be sure to include explicit answers to all questions asked, using the **text** feature of Maple to insert them in the worksheet. Also use the **text** feature of Maple to include your name and section number at the top of the worksheet (do NOT write any of this material in by hand). Use the editing capabilities of Maple to remove from the worksheet any extraneous material such as any errors you have made or cries for help.

Problems

1. It is said that the first thing that everyone does when introduced to Maple is to compute many digits of π . To ask for 100 decimal digits, you need only enter `evalf(Pi, 100);`. Computations like this are based on formulas that show that numbers like π differ by an amount known to be small from something that can be found exactly. For example, the Maclaurin series for $\arctan(x)$ converges sufficiently quickly that about 70 terms suffice to find $\arctan(1/5)$ to 100 decimal places. This led Machin, in 1706, to compute π to 100 decimal places using the formula

$$\pi/4 = 4 \arctan(1/5) - \arctan(1/239).$$

Duplicate his effort using the Maple commands `piOver4:=4*arctan(1/5) - arctan(1/239);` (which shows that Maple will accept unusual names for variables) and `evalf(4*piOver4);`. Note the use of `*` when two terms are multiplied.

Although, Maple will do this type of computation, its real strength is in symbolic work. It can **prove** Machin’s formula. To do this, enter `expand(tan(piOver4));`. This value is exact: to see how it was done, look at `expand(tan(A+B));`. All that is needed to complete the proof is a very crude estimate on the value we called `piOver4`. Why is this? (Use **text** to explain why x is not determined from $\tan(x)$ although we claim that the arctangent is an inverse function.

2. Use Maple’s `simplify` command to simplify the expression

$$\frac{9x^2y - 4y^3}{3x^3y + x^2y^2 - 2xy^3}.$$

Introduce a variable h to represent the expression. After you look at the simplified form by saying `simplify(h);` take another look at h . Has it changed? (Answer in **text**). Repeat the `simplify` command, assigning the result to a new variable. Both variables will be used in the next problem.

3. Use Maple’s `subs` command to evaluate the two variables of Problem 2 at the point $x = 2, y = 3$. Explain the difference (using **text**).

4. Use Maple's `plot` command to determine if the function x^x has any local maxima or minima on the interval $0 \leq x \leq 4$. The instruction `plot(x^x, x=0..4);` gives a graph that shows some features of the function on the given interval, but is useless for studying local maxima or minima. To get a more useful graph, you can also restrict the second coordinate. The instruction `plot(x^x, x=0..4, y=0..10);` limits the range shown and also introduces the label y for the vertical axis. Then choose ranges for both variables to give one more graph that illustrates a smaller region that is more useful for studying local maxima or minima. (Note: x^x is not defined for $x = 0$, but except MapleV, release 5, this has caused no difficulty.)
5. Find the first, second, and third derivatives of the expression $h = e^{x^2}$. (First define an expression for h and then repeatedly use Maple's `diff` command to introduce new named expressions for the higher derivatives.) Form a **list** of these expressions and use the `subs` command to evaluate these derivatives at $x = 0$. Some of these terms are zero. Do you think this is the beginning of a pattern? (Give a brief **text** answer, indicating other things that might be done to test for a pattern.)
6. Use `plot({x, cos(x)}, x=0..1);` to graph $y = x$ and $y = \cos(x)$ on the same set of axes. The graphs intersect at a point where $x = y = \cos(x)$. Use Maple's `fsolve` command to find the x coordinate of this point. Check by finding the cosine of the result.
7. Use Maple's `int` command to find the indefinite integral $\int x^2 \cos(x) dx$ and the definite integral $\int_0^\pi x^2 \cos(x) dx$.
8. Maple's `plot` command can also be used to plot parametric equations. For example, the command `plot([2*sin(t), 2*cos(t), t=-Pi..Pi]);` plots a circle of radius 2. Use this feature to plot (in your worksheet) the equations

$$x(t) = \cos(t)(1 - 2 \sin(3t)), \quad y(t) = \sin(t)(1 - 2 \sin(3t)) \quad \text{for } 0 \leq t \leq 2\pi.$$

9. To obtain special plots, it is first necessary to issue the Maple command `with(plots):`. Do so, then `implicitplot(2*ln(F) - 1.2*F = -ln(R) + 0.9*R - 2.1, F=0..3, R=0..3);` to graph an exact solution of the system of differential equations

$$\begin{aligned} \frac{dR}{dt} &= 2R - 1.2RF \\ \frac{dF}{dt} &= -F + 0.9RF \end{aligned}$$

with the initial conditions $R = F = 1$. Then (in **text**) estimate the largest values of the variables R and F on this curve.

10. Use the Maple command `plot3d` to obtain a plot (in your worksheet) of

$$z = y(1 - 10xy)e^{-x^2 - y^2}$$

over the range $-3 \leq x \leq 3$, $-3 \leq y \leq 3$. (This requires the **plots** package, but Maple remembers that you loaded it in the previous problem.) Rotate the plot until you get a good view of the surface. Then use the **Context Bar** (or menu) to change the **Axes** option to **Boxed** to get a better idea of the function values. Rotate the plot again until you are able to estimate (to about one decimal place) the maximum value of the function. Enter this value into your Maple worksheet (with a sentence stating what it is) using the **text** feature of Maple.