Sep. 20, 2011 QUIZ, MATH 251–13,14,15 Name: ______________________

Signature: ______________________

1. Consider the lines

$$\begin{cases} x(t) = 4 - 2t \\ y(t) = 5 + t \\ z(t) = 2 - t \end{cases} \quad \text{and} \quad \begin{cases} x(t) = 2 + t \\ y(t) = 2 \\ z(t) = 6 + 3t \end{cases}$$

(a) Find a vector perpendicular to the directions of both lines.

Solution: *The vector* $\overrightarrow{A} = \langle -2, 1, -1 \rangle$ *points in the direction of the first line, and the vector* $\overrightarrow{B} = \langle 1, 0, 3 \rangle$ *points in the direction of the second line. One correct answer is*

$$\overrightarrow{A} \times \overrightarrow{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 0 & 3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} -2 & -1 \\ 1 & 3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} -2 & 1 \\ 1 & 0 \end{vmatrix} \mathbf{k} = 3\mathbf{i} + 5\mathbf{j} - \mathbf{k}.$$

(b) Find an equation of a plane containing the first line and parallel to the second line.

Solution: *For a normal vector, we need a vector perpendicular to the directions of both lines, so we can use the answer to (a). The equation will therefore have the form* $3x + 5y - z = c$ *for some constant* c*. We also need a point on the plane; any point on the first line will do, such as* $(4, 5, 2)$ *(corresponding to* $t = 0$*). Therefore* $c = 3 \cdot 4 + 5 \cdot 5 - 2 = 35$ *and the desired plane is*

$$3x + 5y - z = 35.$$

2. In the xy-plane the equation $y = 2x$ determines a line through the origin. In the yz-plane the equation $y = 2z$ determines a line through the origin. Considering both lines in 3-space, what is the angle between the two lines? You may leave your answer in terms of inverse trig functions.

Solution: *The vector* $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$ *points along the first line, and the vector* $\mathbf{w} = 2\mathbf{j} + \mathbf{k}$ *points along the second line. The angle* θ *between them is*

$$\theta = \arccos \frac{\mathbf{v} \bullet \mathbf{w}}{|\mathbf{v}||\mathbf{w}|} = \arccos \frac{4}{\sqrt{5}\sqrt{5}} = \arccos \frac{4}{5}.$$

Sep. 20, 2001 QUIZ, MATH 251–13,14,15 Name: ____________________

Signature: ____________________

1. Consider the lines

$$\begin{cases} x(t) = 2 - t \\ y(t) = 3 + t \\ z(t) = 4 - 2t \end{cases} \quad \text{and} \quad \begin{cases} x(t) = 6 + 3t \\ y(t) = 1 \\ z(t) = 2 + t \end{cases}$$

(a) Find a vector perpendicular to the directions of both lines.

Solution: *The vector* $\overrightarrow{A} = \langle -1, 1, -2 \rangle$ *points in the direction of the first line, and the vector* $\overrightarrow{B} = \langle 3, 0, 1 \rangle$ *points in the direction of the second line. One correct answer is*

$$\overrightarrow{A} \times \overrightarrow{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & -2 \\ 3 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} -1 & 1 \\ 3 & 0 \end{vmatrix} \mathbf{k} = \mathbf{i} - 5\mathbf{j} - 3\mathbf{k}.$$

(b) Find an equation of a plane parallel to the first line and containing the second line.

Solution: *For a normal vector, we need a vector perpendicular to the directions of both lines, so we can use the answer to (a). The equation will therefore have the form* $x - 5y - 3z = c$ *for some constant* c*. We also need a point on the plane; any point on the second line will do, such as* $(6, 1, 2)$ *(corresponding to* $t = 0$*). Therefore* $c = 6 - 5 \cdot 1 - 3 \cdot 2 = -5$ *and the desired plane is*

$$x - 5y - 3z = -5.$$

2. In the xy-plane the equation $y = 3x$ determines a line through the origin. In the xz-plane the equation $x = 3z$ determines a line through the origin. Considering both lines in 3-space, what is the angle between the two lines? You may leave your answer in terms of inverse trig functions.

Solution: *The vector* $\mathbf{v} = \mathbf{i} + 3\mathbf{j}$ *points along the first line, and the vector* $\mathbf{w} = 3\mathbf{i} + \mathbf{k}$ *points along the second line. The angle* θ *between them is*

$$\theta = \arccos \frac{\mathbf{v} \bullet \mathbf{w}}{|\mathbf{v}||\mathbf{w}|} = \arccos \frac{3}{\sqrt{10}\sqrt{10}} = \arccos \frac{3}{10}.$$