Math 251–13–14–15 (CIRCLE YOUR SECTION) Name: ____________________

no-credit QUIZ, September 27, 2001 Signature: ____________________

1. Interpret the function $\mathbf{r}(t) = e^t\mathbf{i} + e^{-t}\mathbf{j}$ as the position function of a moving particle, and compute the quantities in (a)–(e) as functions of t:

(a) the velocity

Solution: $\mathbf{r}'(t) = e^t\mathbf{i} - e^{-t}\mathbf{j}$

(b) the speed

Solution: $v(t) = \sqrt{e^{2t} + e^{-2t}}$

(c) the acceleration

Solution: $\mathbf{r}''(t) = e^t\mathbf{i} + e^{-t}\mathbf{j}$

(d) the curvature

Solution:

$$\mathbf{r}' \times \mathbf{r}'' = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ e^t & -e^{-t} & 0 \\ e^t & e^{-t} & 0 \end{vmatrix} = 2\mathbf{k},$$

so $\kappa = |\mathbf{r}' \times \mathbf{r}''|/|v|^3 = 2/(e^{2t} + e^{-2t})^{3/2}$

(e) the normal component of acceleration

Solution: $a_{nor} = \kappa v^2 = 2/\sqrt{e^{2t} + e^{-2t}}$

(f) Give (but do not evaluate) an integral representing the length of the path from $t = 0$ to $t = 2$.

Solution:

$$\int_0^2 \sqrt{e^{2t} + e^{-2t}}\, dt$$

(g) Is the curve smooth at every point? Why?

Solution: *Yes, because* $\mathbf{r}'(t) = e^t\mathbf{i} - e^{-t}\mathbf{j}$ *is not equal to zero for any* t*.*