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1. Find z_{xy} if $z = \ln(x^2 + y^2)$.

Solution: $z_x = \dfrac{2x}{x^2+y^2}$, $z_{xy} = (z_x)_y = \dfrac{0-(2x)(2y)}{(x^2+y^2)^2} = \dfrac{-4xy}{(x^2+y^2)^2}$.

2. Find an equation for the plane tangent to $z = \dfrac{x}{2x+y}$ at the point $(4, -6, 2)$.

Solution: $z_x = \dfrac{2x+y-2x}{(2x+y)^2} = \dfrac{y}{(2x+y)^2}$ *and* $z_y = \dfrac{-x}{(2x+y)^2}$. *So*

$$\mathbf{N} = -z_x\Big|_{(4,-6,2)}\mathbf{i} - z_y\Big|_{(4,-6,2)}\mathbf{j} + \mathbf{k} = -\frac{-6}{2^2}\mathbf{i} - \frac{-4}{2^2}\mathbf{j} + \mathbf{k} = (3/2)\mathbf{i} + \mathbf{j} + \mathbf{k}$$

and the plane therefore has equation

$$(3/2)(x-4) + (y+6) + (z-2) = 0.$$

3. If $A^2 + B^2 + C^2 + AB + AC + BC = 0$, find $\dfrac{\partial A}{\partial B}$.

Solution: *Method 1: Let* $F(A, B, C) = A^2 + B^2 + C^2 + AB + AC + BC$. *Then*

$$\frac{\partial A}{\partial B} = -\frac{\partial F/\partial B}{\partial F/\partial A} = -\frac{2B+A+C}{2A+B+C}.$$

Method 2: Implicit differentiation—regard A as a function of the independent variables B and C, and differentiate the equation with respect to B, holding C constant:

$$2A\frac{\partial A}{\partial B} + 2B + \frac{\partial A}{\partial B}B + A + \frac{\partial A}{\partial B}C + C = 0.$$

Solving for $\dfrac{\partial A}{\partial B}$ *yields the same answer.*