Math 251–13–14–15 (`CIRCLE YOUR SECTION`) SOLUTIONS, version 1

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1. Find $\iint_R y\,dA$, where R is the rainbow-shaped region above the x-axis and between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Solution: The region R is described in polar coordinates by $1 \le r \le 2$ and $0 \le \theta \le \pi$. Also $y = r\sin\theta$, and the Jacobian is r (as always for polar coordinates). So

$$\iint_R y\,dA = \int_{r=1}^{2}\int_{\theta=0}^{\pi}(r\sin\theta)r\,d\theta dr = \int_{r=1}^{2}\int_{\theta=0}^{\pi} r^2\sin\theta\,d\theta dr$$

$$\text{(by the rectangle trick)} = \int_{r=1}^{2} r^2\,dr \int_{\theta=0}^{\pi}\sin\theta\,d\theta = \left(\frac{8}{3} - \frac{1}{3}\right)\cdot(1+1) = \frac{14}{3}.$$

The rectangle trick applies because (1) limits of integration are all constants, and (2) the integrand is the product of a function of r and a function of θ.

2. Sketch the region R in the first quadrant enclosed by x-axis, the line $y = 2x$ and the curve $y = x^2 - 8$. Then set up

$$\iint_R xy\,dA$$

as an iterated integral. Do not attempt to evaluate this integral.

Solution: (Sketch not shown.) The curves $y = 2x$ and $y = x^2 - 8$ meet where $2x = x^2 - 8$; in the first quadrant this is at $(x, y) = (4, 8)$. The region is type II, with bottom line $y = 0$, top line $y = 8$; left boundary $y = 2x$ (i.e. $x = y/2$) and right boundary $y = x^2 - 8$ (i.e. $x = \sqrt{y+8}$, in the first quadrant). So

$$\iint_R xy\,dA = \int_{y=0}^{8}\left[\int_{x=y/2}^{\sqrt{y+8}} xy\,dx\right]dy.$$