1. Evaluate the double integral $\iint_D x \sin y \, dA$ where D is bounded by $y = 0$, $y = x^2$ and $x = 1$.

Solution: The lower edge of D is the x-axis $y = 0$; the upper edge is $y = x^2$, meeting the lower edge at the origin. The right edge is $x = 1$. So

$$\iint_D x \sin y \, dA = \int_{x=0}^{1} \left[\int_{y=0}^{x^2} x \sin y \, dy \right] dx = \int_{x=0}^{1} \Big[-x \cos y \Big]_{y=0}^{y=x^2} dx$$

$$= \int_{x=0}^{1} (-x \cos x^2 + x) \, dx = -\frac{1}{2} \sin x^2 + \frac{x^2}{2} \Big|_0^1$$

$$= -\frac{1}{2} \sin 1 + \frac{1}{2} = \frac{1 - \sin 1}{2}.$$

2. Set up a double integral to find the volume of the top half-ball bounded by the plane $z = 0$ and the half-sphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$.

Solution: The half-ball lies above the disk $D : x^2 + y^2 \leq 1$ in the x, y-plane, and under the half-sphere $z = \sqrt{1 - x^2 - y^2}$. The volume is

$$\iint_D \sqrt{1 - x^2 - y^2} \, dA = \int_{x=-1}^{1} \int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1 - x^2 - y^2} \, dy \, dx.$$

(The integral is best evaluated by polar coordinates. Without polar coordinates it can be evaluated by using the inverse trig substitution $y = \sqrt{1 - x^2} \sin u$ in the inner integral.)