1. Evaluate the triple integral $\iiint\limits_E x^2 + y^2 + z^2 \, dV$ where E is the solid bounded below by the cone $\phi = \pi/4$ and above by the sphere $\rho = 2$.

Solution: The right circular cone $\phi = \pi/4$ is "inverted", having its vertex at the origin. Its axis of symmetry is the positive z-axis. Its sides slope at 45°, that is, it is obtained by revolving the line $z = x$ (or $z = y$) about the z-axis. The solid E is described by $0 \le \phi \le \pi/4$, $0 \le \rho \le 2$, $0 \le \theta \le 2\pi$.

$$\begin{aligned}
\iiint\limits_E x^2 + y^2 + z^2 \, dV &= \int_{\phi=0}^{\pi/4} \int_{\rho=0}^{2} \int_{\theta=0}^{2\pi} \rho^2 (\rho^2 \sin\phi) \, d\theta \, d\rho \, d\phi \\
&= 2\pi \int_{\phi=0}^{\pi/4} \int_{\rho=0}^{2} \rho^4 \sin\phi \, d\rho \, d\phi \\
&= 2\pi \int_{\phi=0}^{\pi/4} \frac{2^5}{5} \sin\phi \, d\phi \\
&= -\frac{64}{5} \cos\phi \Big|_0^{\pi/4} = \frac{64}{5} \left(1 - \frac{\sqrt{2}}{2}\right).
\end{aligned}$$