REVIEW PROBLEMS FOR EXAM #1, 251-13-14-15, FALL 2001

The exam will cover the syllabus from Section 12.1 through Section 14.5. You will be allowed to use a calculator without symbolic capabilities, such as TI-82, 83, 85, 86. You may not bring a crib sheet. You will be given a short list of formulas with the exam (see next page).
The following problems are mostly taken or adapted from recent 251 exams.

1. Find an equation of the plane which passes through the points $P(1,4,6)$, $Q(-2,5,-1)$ and $R(1,-1,1)$. Also find the area of the triangle with these vertices.

2. Find the length of one complete turn of the helix $\mathbf{r}(t) = 2\sin 2t\,\mathbf{i} + 2\cos 2t\,\mathbf{j} + 3t\,\mathbf{k}$.

3. Find parametric equations for the line passing through the point $(2,-7,5)$ which is parallel to the line $x = 2t+7$, $y = -3t+5$, $z = t+1$.

4. The three vectors $\mathbf{i}+\mathbf{j}$, $\mathbf{j}+2\mathbf{k}$ and $\mathbf{i}-\mathbf{k}$ (with tails at the origin) are the edges of a parallelpiped. Find the volume of the parallelepiped.

5. Find an equation of the tangent plane to the surface $z = \sin(x^2+y)$ at the point $(1,-1,0)$.

6. Find the tangential and normal components of acceleration for a particle moving with position function

$$\mathbf{r}(t) = (t-\sin t)\,\mathbf{i} - (1-\cos t)\,\mathbf{j}, \quad 0 \le t \le 2\pi.$$

7. Find the curvature of the curve $y = e^x$ (as a function of x). Also find the point on the curve where the curvature is maximum.

8. If $xyz = \sin(x^2+2y+z)$, find $\dfrac{\partial z}{\partial x}$.

9. Find the equation of the tangent plane to the surface $xy+yz+xz = 11$ at the point $(1,2,3)$.

10. Use vectors to find the angle between the main diagonal of a cube and the diagonal of one of its faces (both diagonals emerging from a common vertex).

11. Find the velocity and position of a particle that starts at the origin at time $t=0$ with velocity $\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and has acceleration $\mathbf{a}(t) = t\,\mathbf{i}+\mathbf{j}+t^2\,\mathbf{k}$.

12. For the curve $\mathbf{r}(t) = (t^3/3)\,\mathbf{i} + (t^2/2)\,\mathbf{j} + t\,\mathbf{k}$, find the unit tangent vector and the curvature (as functions of t).

13. Find the angle between the planes $x+y+2z=1$ and $x-2y+z=1$, and also find equation(s) for the line of intersection of those planes.

14. Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x+2y)^2}{x^2+y^2}$ or prove that the limit doesn't exist.

15. The gravitational force on a meteor has magnitude $F = cm/(h+R)^2$, where R is the radius of the earth, c is a constant, m is the mass of the meteor (which decreases as the meteor burns up), and h is its height above the surface of the earth. Find a formula relating $\dfrac{dF}{dt}$ to $\dfrac{dm}{dt}$ and $\dfrac{dh}{dt}$ (and perhaps other quantities too).

16. If $z = x^2\sin y$, $x = s^2+t^2$ and $y = 2st$, find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.

17. If $z = f(x,y)$ and $x = u+v$ and $y = u-v$, show that

$$\left(\frac{\partial z}{\partial u}\right)^2 - \left(\frac{\partial z}{\partial v}\right)^2 = 4\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}.$$

18. Find dz if $z = xe^{(y/x)}$.

19. Find two parallel planes, one containing the line $\mathbf{r}_1(t) = \langle 1,0,0\rangle + t\langle 1,1,1\rangle$ and the other containing the line $\mathbf{r}_2(t) = \langle 1,1,0\rangle + t\langle 0,1,1\rangle$.

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Formula List

$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

$comp_{\mathbf{a}}(\mathbf{b}) = \frac{\mathbf{a} \bullet \mathbf{b}}{|\mathbf{b}|}$

$proj_{\mathbf{a}}(\mathbf{b}) = \frac{\mathbf{a} \bullet \mathbf{b}}{\mathbf{b} \bullet \mathbf{b}} \mathbf{b}$

$$\kappa = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{v^3} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{\left|\frac{d\mathbf{T}}{dt}\right|}{v}$$

$$\frac{ds}{dt} = v = |\mathbf{r}'(t)|$$

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