REVIEW PROBLEMS FOR EXAM #2, 251-13-14-15, FALL 2001

The exam will cover the syllabus from Section 14.6 through Section 16.2 (and including 12.7). You will be allowed to use a calculator without symbolic capabilities, such as TI-82, 83, 85, 86. You may not bring a crib sheet. You will be given a short list of formulas with the exam (see next page).

The following problems are mostly taken or adapted from recent 251 exams. (Answers are not guaranteed.)

1. Find the directional derivative of $f(x,y,z) = \cos(xy) + e^{yz} - \ln(xz)$ at $(1,0,2)$ in the direction from $(1,0,2)$ toward $(2,3,4)$. What direction gives the largest directional derivative of this function at $(1,0,2)$, and what is the value of that largest directional derivative? Ans. $4/\sqrt{14}$, $\langle -1, 2, -1/2\rangle$, $\sqrt{21}/2$

2. Find equations for the tangent plane and normal line to the surface $x^4y + y^4z + 2z^4x = 4$ at the point $(1,1,1)$. Ans. Tang. plane: $6x + 5y + 9z = 20$.

3. Find the absolute maximum and minimum values of the function $f(x,y,z) = x - 2y + 3z$ on the ellipsoid $x^2 + 2y^2 + 3z^2 = 24$. Ans. 12 and -12.

4. Find all the critical points of $f(x,y) = x^3 - 6xy + y^3$, and classify them as local maxima, local minima, or saddle points. Ans. $(0,0)$ saddle; $(2,2)$ local min.

5. Find ∇f if $f(x,y) = \dfrac{y - x^2}{4}$, and sketch the vector field ∇f. Ans. $(-x/2)\,\boldsymbol{i} + (1/4)\,\boldsymbol{j}$.

6. For the function $f(x,y) = x^2 - xy + y^2 - 3y$, find the absolute maximum and minimum values on the triangular region bounded by the lines $x = 0$, $y = 4$, and $y = x$.
Ans. min -3 at $(1,2)$, max 4 at $(0,4)$ and $(4,4)$

7. Compute $\displaystyle\iint_R 12x\,dA$ where R is the triangle in the second quadrant $(x \le 0, y \ge 0)$ enclosed by the coordinate axes and the line $2x - y + 2 = 0$. Ans. -4

8. Sketch the region of integration for the integral $\displaystyle\int_0^1 \int_{3x^2}^{3x} 2xy\,dy\,dx$. Write an equivalent integral with the order of integration reversed. Evaluate both integrals and check that they are equal.
Ans. $\int_0^3 \int_{y/3}^{\sqrt{y/3}} 2xy\,dx\,dy = 3/4$.

9. Find the centroid of the region bounded by the y-axis and the parabola $y = 1 - x^2$. Ans. $(0, 2/5)$

10. Evaluate $\displaystyle\int_{x=0}^{2} \int_{y=0}^{\sqrt{4-x^2}} xy^2\,dy\,dx$ by using polar coordinates. Ans. $32/15$

11. Evaluate $\displaystyle\int_{x=-2}^{2} \int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} e^{-x^2-y^2}\,dy\,dx$ by using polar coordinates. Ans. $\pi(1 - e^{-4})$

12. Consider the integral $\displaystyle\iiint_S (x^2 + y^2)\,dV$ where S is that part of the ball $x^2 + y^2 + z^2 \le 9$ in the first octant. Set up the integral in two ways, first using cylindrical coordinates and then using spherical coordinates. Use one of these two to evaluate the integral. Ans. $81\pi/5$

13. Evaluate $\displaystyle\iiint_S y\,dV$ where S is the solid in the first octant cut off by the plane $x + 2y + 2z = 4$.
Ans. $4/3$

14. Compute $\int_C xy\,dx + y^2\,dy$ where C is the quarter-circle from $(2,0)$ to $(0,2)$ (centered at the origin).
Ans. 0

15. Compute $\int_C xy\,dx + y^2\,dy$ where C is the straight line segment from $(2,0)$ to $(0,2)$. Ans. $4/3$

16. Compute $\int_C (x + 2y)\,ds$ where C is the straight line segment from $(2,0)$ to $(0,2)$. Ans. $6\sqrt{2}$

17. Find the Jacobian of the transformation $x = 2u$, $y = 3v$. Use this transformation as a first step to compute $\displaystyle\iint_R x^2\,dA$ where R is the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} \le 1$. Ans. 6, and 6π.

OVER

$$\left|\frac{\partial(x,y)}{\partial(r,\theta)}\right| = r$$

$$\left|\frac{\partial(x,y,z)}{\partial(r,\theta,z)}\right| = r$$

$$\left|\frac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)}\right| = \rho^2 \sin\phi$$

$$\iint\limits_{R} f(x,y)\,dx\,dy = \iint\limits_{R'} f\big(x(u,v),y(u,v)\big)\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,du\,dv$$

$$x = \rho\sin\phi\cos\theta,\ y = \rho\sin\phi\sin\theta,\ z = \rho\cos\phi$$

$$D_{\boldsymbol{u}}f(x_0,y_0) = \nabla f(x_0,y_0) \bullet \boldsymbol{u}$$