REVIEW PROBLEMS FOR EXAM #2, 251-13-14-15, FALL 2001

further notes on problems 12 and 13

12. In cylindrical coordinates it's

$$\int_{\theta=0}^{\pi/2}\int_{r=0}^{3}\int_{z=0}^{\sqrt{9-r^2}} r^3\,dz\,dr\,d\theta = \int_{\theta=0}^{\pi/2}\int_{r=0}^{3} r^3\sqrt{9-r^2}\,dr\,d\theta.$$

Substitute $u = 9 - r^2$, so that $r^3\sqrt{9-r^2}\,dr = r^2 \cdot r = (9-u)(-1/2)\sqrt{u}\,du$, etc., etc.

In spherical coordinates it's $\displaystyle\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{\pi/2}\int_{\rho=0}^{3} \rho^4 \sin^3\phi\,d\rho\,d\phi\,d\theta.$

13. This can be done in any one of the six orders. For example,

$$\begin{aligned}\int_{y=0}^{2}\int_{x=0}^{4-2y}\int_{z=0}^{(4-x-2y)/2} y\,dz\,dx\,dy &= \int_{y=0}^{2}\int_{x=0}^{4-2y} y(4-x-2y)/2\,dx\,dy \\ &= \int_{y=0}^{2} y\left[\frac{(4-2y)x}{2} - \frac{x^2}{4}\right]_{x=0}^{x=4-2y} dy \\ &= \int_{y=0}^{2} y\frac{(4-2y)^2}{4}\,dy = \frac{4}{3}\end{aligned}$$