1. A solid object in the first octant is bounded by the surfaces $z = 1-y^2$, $z = 1-x$, $x = 0$, and $y = 0$. Its (variable) density is $\delta(x, y, z) = 30y$. Find its mass.

SOLUTION: Draw the part of the line $z = 1 - x$ in the first quadrant of the x, z-plane (we don't care about the rest of the line). The **plane** $z = 1 - x$ contains this line and is parallel to the y-axis.

Draw the part of the parabola $z = 1 - y^2$ in the first quadrant of the y, z-plane (we don't care about the rest of the parabola). The **surface** $z = 1 - y^2$ is formed by lines meeting this parabola and parallel to the x-axis. This surface is a "parabolic cylinder".

Either the x-axis or the y-axis can be thought of as the "floor-to-ceiling" axis of the given solid S.

Solution 1: If the x-axis is thought of as the "floor-to-ceiling" axis, then the floor is the region in the first quadrant of the y, z-plane cut off by $z = 1-y^2$, the ceiling is part of the plane $z = 1 - x$, and the "sides" are formed by the surface $z = 1 - y^2$, the x, y-plane and the x, z-plane. This way,

$$m = \iiint_S \delta\, dV = \int_{y=0}^{1} \int_{z=0}^{1-y^2} \int_{x=0}^{1-z} 30y\, dx\, dz\, dy = \int_{y=0}^{1} \int_{z=0}^{1-y^2} 30xy \Big|_{x=0}^{x=1-z} dz\, dy$$

$$= \int_{y=0}^{1} \int_{z=0}^{1-y^2} 30y(1-z) \Big|_{x=0}^{x=1-z} dz\, dy = \int_{y=0}^{1} 30y \left(z - \frac{z^2}{2}\right) \Big|_{z=0}^{z=1-y^2} dy$$

$$= \int_{y=0}^{1} 30y \left(1 - y^2 - \frac{(1-y^2)^2}{2}\right) dy = \int_{y=0}^{1} 15(y - y^5)\, dy = 5.$$

Solution 2: If the y-axis is the "floor-to-ceiling" axis then the floor is the triangle in the x, z-plane enclosed by the two axes and $z = 1 - x$. The ceiling is $z = 1 - y^2$ (that is, $y = \sqrt{1-z}$), and the sides are the y, z-plane, the x, y-plane, and the plane $z = 1-x$. This way,

$$m = \iiint_S \delta\, dV = \int_{x=0}^{1} \int_{z=0}^{1-x} \int_{y=0}^{\sqrt{1-z}} 30y\, dy\, dz\, dx = \int_{x=0}^{1} \int_{z=0}^{1-x} 15y^2 \Big|_{y=0}^{y=\sqrt{1-z}} dz\, dx$$

$$= \int_{x=0}^{1} \int_{z=0}^{1-x} 15(1-z)\, dz\, dx = \int_{x=0}^{1} \frac{15}{2}(2z - z^2) \Big|_{z=0}^{z=1-x} dx$$

$$= \frac{15}{2} \int_{x=0}^{1} 2(1-x) - (1-x)^2\, dx = \frac{15}{2} \int_{x=0}^{1} (1-x^2)\, dx = 5$$

.