REVIEW PROBLEMS FOR EXAM #1, 251-04-05-06, FALL 2005

The exam will cover the syllabus from Section 12.1 through Section 14.6. Implicit differentiation, however, will not be included. You will be allowed to use a calculator without symbolic capabilities, such as TI-82, 83, 85, 86. You may not bring a crib sheet.
The following problems are mostly taken or adapted from recent 251 exams.

1. Find an equation of the plane which passes through the points $P(1,4,6)$, $Q(-2,5,-1)$ and $R(1,-1,1)$. Also find the area of the triangle with these vertices.

2. Find the length of one complete turn of the helix $\vec{r}(t) = 2\sin 2t\,\vec{i} + 2\cos 2t\,\vec{j} + 3t\,\vec{k}$.

3. Find parametric equations for the line passing through the point $(2,-7,5)$ which is parallel to the line $x = 2t+7$, $y = -3t+5$, $z = t+1$.

4. The three vectors $\vec{i} + \vec{j}$, $\vec{j} + 2\vec{k}$ and $\vec{i} - \vec{k}$ (with tails at the origin) are the edges of a parallelpiped. Find the volume of the parallelepiped.

5. Find an equation of the tangent plane to the surface $z = \sin(x^2 + y)$ at the point $(1,-1,0)$.

6. Find the tangential and normal components of acceleration for a particle moving with position function

$$\vec{r}(t) = (t - \sin t)\,\vec{i} - (1 - \cos t)\,\vec{j}, \quad 0 \le t \le 2\pi.$$

7. Find the curvature of the curve $y = e^x$ (as a function of x). Also find the point on the curve where the curvature is maximum.

8. If $z = \dfrac{e^{2y}}{(1+x)^2}$, find the unit vector $\vec{u}$ for which $D_{\vec{u}}z(0,0)$ is as large as possible. Also find the directional derivative of z at $(0,0)$ in the direction toward $(\pi, 2\pi)$.

9. Give the equation of the level curve of $z = x^2y$ passing through the point $(3,5)$, and sketch the curve.

10. Use vectors to find the angle between the main diagonal of a cube and the diagonal of one of its faces (both diagonals emerging from a common vertex).
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11. Find the velocity and position of a particle that starts at the origin at time $t = 0$ with velocity $\vec{i} + 2\vec{j} + \vec{k}$ and has acceleration $\vec{a}(t) = t\,\vec{i} + \vec{j} + t^2\,\vec{k}$.

12. For the curve $\vec{r}(t) = (t^3/3)\,\vec{i} + (t^2/2)\,\vec{j} + t\,\vec{k}$, find the unit tangent vector and the curvature (as functions of t).

13. Find the angle between the planes $x + y + 2z = 1$ and $x - 2y + z = 1$, and also find equation(s) for the line of intersection of those planes.

14. Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x+2y)^2}{x^2+y^2}$ or prove that the limit doesn't exist.

15. The gravitational force on a meteor has magnitude $F = cm/(h+R)^2$, where R is the radius of the earth, c is a constant, m is the mass of the meteor (which decreases as the meteor burns up), and h is its height above the surface of the earth. Find a formula relating $\dfrac{dF}{dt}$ to $\dfrac{dm}{dt}$ and $\dfrac{dh}{dt}$ (and perhaps other quantities too).

16. If $z = x^2 \sin y$, $x = s^2 + t^2$ and $y = 2st$, find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.

17. If $z = f(x,y)$ and $x = u^2 - v^2$ and $y = 2uv$, show that there is a constant c such that

$$\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2 = c(u^2+v^2)\left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right]$$

(Use the chain rule.)

18. Find dz if $z = xe^{(y/x)}$.

19. Find two parallel planes, one containing the line $\vec{r}_1(t) = \langle 1,0,0\rangle + t\langle 1,1,1\rangle$ and the other containing the line $\vec{r}_2(t) = \langle 1,1,0\rangle + t\langle 0,1,1\rangle$.

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