REVIEW PROBLEMS FOR EXAM #2, 251-04-05-06, FALL 2005

The exam will cover the material since the previous exam, through Section 16.2. You will be allowed to use a calculator without symbolic capabilities, such as TI-82, 83, 85, 86. You may not bring a crib sheet.

The following problems are mostly taken or adapted from recent 251 exams. (Answers are not guaranteed.)

1. Find equations for the tangent plane and normal line to the surface $x^4y + y^4z + 2z^4x = 4$ at the point $(1, 1, 1)$.
Ans. Tang. plane: $6x + 5y + 9z = 20$.

2. The equation $x^3y\cos z + 2x^2z - 5e^{yz} = 3$ defines z as a function of x and y near the point $(2, 1, 0)$. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ as functions of x, y, and z, and evaluate them at $(2, 1, 0)$.

Ans. At $(2, 1, 0)$, $\partial z/\partial x = -4$ and $\partial z/\partial y = -8/3$.

3. Find the absolute maximum and minimum values of the function $f(x, y, z) = x - 2y + 3z$ on the ellipsoid $x^2 + 2y^2 + 3z^2 = 24$.
Ans. 12 and -12.

4. Find all the critical points of $f(x, y) = x^3 - 6xy + y^3$, and classify them as local maxima, local minima, or saddle points.
Ans. $(0, 0)$ saddle; $(2, 2)$ local min.

5. For the function $f(x, y) = x^2 - xy + y^2 - 3y$, find the absolute maximum and minimum values on the triangular region bounded by the lines $x = 0$, $y = 4$, and $y = x$.

Ans. min -3 at $(1, 2)$, max 4 at $(0, 4)$ and $(4, 4)$

6. Compute $\displaystyle\iint_R 12x\, dA$ where R is the triangle in the second quadrant ($x \le 0, y \ge 0$) enclosed by the coordinate axes and the line $2x - y + 2 = 0$.
Ans. -4

7. Sketch the region of integration for the integral $\displaystyle\int_0^1 \int_{3x^2}^{3x} 2xy\, dy\, dx$. Write an equivalent integral with the order of integration reversed. Evaluate both integrals and check that they are equal.

Ans. $\int_0^3 \int_{y/3}^{\sqrt{y/3}} 2xy\, dx\, dy = 3/4$.

8. Find the center of mass of a uniform plane lamina whose boundary is formed by the y-axis and the parabola $y = 1 - x^2$.
Ans. $(0, 2/5)$

9. Use polar coordinates to find: (a) $\displaystyle\int_{x=0}^{2} \int_{y=0}^{\sqrt{4-x^2}} xy^2\, dy\, dx$ and (b) $\displaystyle\int_{x=-2}^{2} \int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} e^{-2x^2-2y^2}\, dy\, dx$.

Ans. (a) $32/15$ and (b) $\pi(1 - e^{-8})/2$

10. Consider the integral $\displaystyle\iiint_S (x^2 + y^2)\, dV$ where S is that quarter of the ball $x^2 + y^2 + z^2 \le 9$ for which $y \ge 0$ and $z \ge 0$. Set up the integral in two ways, first using cylindrical coordinates and then using spherical coordinates. Evaluate both ways and check that the results are the same.
Ans. $162\pi/5$

11. Evaluate $\displaystyle\iiint_S y\, dV$ where S is the solid in the first octant cut off by the plane $x + 2y + 2z = 4$.

Ans. $4/3$

12. Compute $\int_C xy\, dx + y^2\, dy$ where C is the quarter-circle from $(2, 0)$ to $(0, 2)$ (centered at the origin).
Ans. 0

13. Compute $\int_C xy\, dx + y^2\, dy$ where C is the straight line segment from $(2, 0)$ to $(0, 2)$.
Ans. $4/3$

14. Compute $\int_C (x + 2y)\, ds$ where C is the straight line segment from $(2, 0)$ to $(0, 2)$.
Ans. $6\sqrt{2}$

15. Find the Jacobian of the transformation $x = 2u$, $y = 3v$. Use this transformation as a first step to compute $\displaystyle\iint_R x^2\, dA$ where R is the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} \le 1$.
Ans. 6, and 6π.