Mathematics 251 Maple Lab 0

Introduction to Maple

Fall 2006

This project This lab contains seven problems intended to introduce you to some of the basic features of Maple and to give you practice preparing a Maple worksheet. Many of the Maple instructions are in the **seed file**, but there are several places where you are asked to interpret results. Additional information you need to do use Maple is contained in the handout Instructions for Use of Maple in Mathematics 251. There is also a **Supplementary Worksheet** giving related work to get more out of your work with Maple.

This lab is for practice only: the lab will be graded using the same standards that will be applied to later labs, but the grade will be ignored in computing your grade for the course, Use this lab to learn how to prepare your Maple worksheet. In particular, be sure to include explicit answers to all questions asked, using the **text** feature of Maple to insert them in the worksheet. Also use the **text** feature of Maple to include your name and section number at the top of the worksheet (do **NOT** write any of this material in by hand). Use the editing capabilities of Maple to remove any extraneous material from the worksheet .

The seed file begins with the line: "Put header here." That line is to be **replaced** with a header that includes your name and any other information requested by your instructor. This is followed by a **Section 0** that contains global settings used in the worksheet. For this lab, the section contains the line

```
with(plots):with(VectorCalculus):
```

that loads the **plots and VectorCalculus libraries**. Each `with` command ends with a colon to hide the list of all functions in the library that is the normal output of this command. Several lines beginning, "Warning, . . . " may be printed. These are harmless, and may be ignored.

Some of the problems refer to tools in the Student[CalculusI] package. The supplementary worksheet contains examples of these tools. During your session in the IML, you may concentrate on the contents of the main worksheet, but you should download the supplementary worksheet and follow its suggestions as you prepare your report. For subsequent labs, you should obtain both the main seed file and the supplementary worksheet when you begin your work.

Problem 1: Pi It is said that the first thing that everyone does when introduced to Maple is to compute many digits of π. Begin by asking for 100 decimal digits with the command `evalf[100](Pi);` (in the seed file).

Various formulas approximate π by something that can be found exactly, with a known bound on the difference. For example, the Maclaurin series for $\arctan(x)$ converges sufficiently quickly that you can be sure that 70 terms suffice to find $\arctan(1/5)$ to 100 decimal places, and 21 terms will give $\arctan(1/239)$ to the same accuracy. This led Machin, in 1706, to compute π to 100 decimal places using the formula

$$\frac{\pi}{4} = 4\arctan\frac{1}{5} - \arctan\frac{1}{239}.$$

Duplicate part of his effort (ignoring the details of how the arctangent is computed) using the Maple commands `MpiOver4:=4*arctan(1/5) - arctan(1/239);` (which shows that Maple accepts unusual names for variables), and `evalf[100](4*MpiOver4);` . Note the use of `*` to denote multiplication.

Examine these two results: they should look the same. Add a **text comment** confirming that you tested Machin's formula.

Although Maple will do this type of **numerical** computation, its real strength is in **symbolic** work. It can **prove** Machin's formula. To do this, enter `expand(tan(MpiOver4));` . This value is exact: to see how it was done, refresh your memory of a basic formula by looking at `expand(tan(A+B));` (the supplementary worksheet contains details of the application of this identity to Machin's formula). Follow this with a **text comment** describing how repeated use of this formula can expand `tan(MpiOver4)` (space has been left in the seed file for comments requested in this project description).

Actually, a little more is needed because the $\arctan(x)$ selects the solution of $\tan(u) = x$ with $-\pi/2 < u < \pi/2$, so you also need to check that Machin's value is in this interval. Crude estimates on this value are enough to check this. Instead of doing this, the supplementary worksheet contains simple example where the arctangent **does not** give the intended value. The example evaluates `expand(tan(2*arctan(2)));` (which returns the value $-4/3$), and then compares

`V1:=2*arctan(2.);` , and `V2:=arctan(-4./3);` .

Additional computations are suggested to shed more light on this example. When you are satisfied that you know the difference between the values of `V1` and `V2`, add a **text comment** to **explain the difference** to your main worksheet. In particular, you should recognize $V1 - V2$. **Include your interpretation** of this value and its significance in your comment.

Problem 2: Evaluation and Simplification

Consider the expression

$$\frac{2x^3 - 7x^2y + 5xy^2}{2x^3 - 7x^2y + 7xy^2 - 5y^3}.$$

Write a command assigning the Maple version of this expression to the **name** `express2`.

To test your definition, the supplementary worksheet contains instructions to evaluate `express2` at $(1, 0)$, $(0, 1)$, $(1, 1)$, and $(1, -1)$. Note how a **list** is used to pass the values of more than one variable to the `eval` command. These values are easy to obtain, so they can be used to check that you have written the correct Maple expression.

A typical use of the supplementary worksheet is to test expressions that you have constructed. When you are confident that your expression passes appropriate tests, **copy the definition of the expression** to the main worksheet and **execute** the definition by hitting the **Enter** key with the cursor anywhere on the definition.

The supplementary worksheet will give more details of the use of the `simplify` command outlined below.

Try to find the value of this expression at $(x, y) = (5, 2)$. It should give an error.

If you now say `simplify(express2);` the **result** should **look simpler** than the original expression. However, if you give the command `express2;` again to see what that name represents after executing the `simplify` command, you should find that **it has not changed**. In other words, the `simplify` command **shows you the simplification**, but **it does not modify its argument**. When you give the result to a new name (both the supplementary worksheet and the seed file contain the line `express2s:=simplify(express2);` to do this), you will have names for both the original and the simplified expression so that they can be compared.

You can test that these expressions usually evaluate to the same answers by **Copying** the `eval` commands to a new line in the supplementary worksheet and changing `express2` to `express2s`. If there was a value for the original expression, you should get the same value for the simplified expression. However, `express2s` now has a value at $(x, y) = (5, 2)$.

Add a **text comment** to the main worksheet giving what you think is the relation between the original expression and its simplification. This comment may refer to results of this section, or any other Maple instructions that you use to support your claims.

Problem 3: Plotting a function

In this part, we use Maple's `plot` command to determine if the function x^x has any local maxima or minima on the interval $0 \le x \le 4$. The supplementary worksheet contains a line `x^x;` to obtain create this expression **as output**. You can select this output and use the right mouse button to bring up a **context menu**. The menu will contain some options that lead to a plot of this function. When you are preparing your report, experiment with different choices to see how they can help construct plots. The "2-D plot" choice gives a `smartplot` that allows adjustments from a context menu, but doesn't allow you to recover your choices. The "Plot Builder" only allows you to choose the domain in its initial dialog, but the "Options" button gives you a second dialog that allows more choices. When you are done, you see the plot command that you have constructed as well as the graph. Unfortunately, the Plot Builder doesn't remember your previous choices, so new plots must be built afresh.

You should do more experiments in the supplementary worksheet. Using the Plot Builder, or examples from the **Help page** for the `plot` command, construct the instruction to plot $y = x^x$ on the interval $0 < x < 4$ with no restriction on y. Add a **Title** such as "My first plot", and copy this command to the main worksheet.

This plot will not be helpful for determining any local maxima or minima, but it should show a large part of the graph that **does not** contain such points. Return to the supplementary worksheet and modify the plot instruction to narrow the domain and range to a region that may contain relative extrema.

When you have found a graph that shows this feature, copy the instruction to the main worksheet and **add a suitable title**. There will be two graphs in your main worksheet, and only you will know how many others are in the supplementary worksheet.

Add a text comment interpreting your second graph. The comment **should indicate** whether the critical point is a local minimum or a local maximum, and **give its coordinates** to **two decimal places**. (By using Calculus, Maple can find the location of the critical point **exactly**, but that is not part of this project. One method is to use the `ExtremePoints` function in the **Student[Calculus1] package**, that you can load by including a line `with(Student[Calculus1]):` — note the colon — in a worksheet. Details are in the supplementary worksheet. This is entirely optional: no reference to it need appear in the main worksheet.

Problem 4: Derivatives

Find the first, second, and third derivatives of the expression e^{x^2}.

First, write the **Maple input** for this expression (using the `exp` function, **not** the `^` operator, and **assign** the result to the name `Exsq`. (Selecting the exponential from the palette will generate the correct expression.)

Then repeatedly use Maple's `diff` command to introduce new named expressions for the higher derivatives. The names `Exsq1`, `Exsq2`, and `Exsq3` are suggested because they are descriptive, but you may use any names.

In your report, you should collect these into a **list** — consult **Maple Help**, if necessary, to see how to do this — and assign this list to a named variable.

The values of these derivatives at $x = 0$ are related to the series for this function by **Taylor's theorem**.

Use the `eval` command applied to the **name of the list** that you just constructed to find a list of these values. Some of the terms should be zero. Do you think this is the beginning of a pattern? Use a **text comment** to suggest a pattern and identify other things that might be done to test for a pattern. Some suggestions are in the supplementary worksheet.

Problem 5: Integrals

Use Maple's `int` command to find the **indefinite integral** $\int x^2 \cos(x)\, dx$ and the **definite integral** $\int_0^{\pi} x^2 \cos(x)\, dx$. If you want to step through this integration, use the suggestions in the supplementary worksheet.

When you were first learning **techniques of integration**, you were discouraged from attempting to find

$$\int e^{-x^2}\, dx.$$

Since the integrand is continuous, **the integral certainly exists**, but it **cannot be expressed in terms of the familiar functions of calculus**. Maple knows how to find this integral, because it knows about the **new function that was invented** to express this integral. The seed file contains the instructions `int( exp(-x^2), x );` for finding the indefinite integral, and `int( exp(-x^2), x =-infinity .. infinity );` to illustrate how certain **improper integrals** are written in Maple.

Note that Maple doesn't write the $+C$ that you were told to use to indicate that you were finished computing an integral. If you want to define a function as an integral, it is necessary to specify the value of the function at some **base point** — we have a preference for functions that satisfy $f(0) = 0$, although there are exceptions.

Identify the function that appears in the formula for the indefinite integral and look it up in Help. In text, answer the following questions in **text**:

- What name does Maple use for the function?
- The function at x is defined by an integral with x as one endpoint and the other constant. What is that constant, and what does this mean about the value of the function at that value of x?
- What is the role of the scale factor relating this function to the integral appearing in its definition?

Problem 6: Parametric curves

Maple's `plot` command can also be used to plot parametric equations. The **astroid** is defined by

$$x = \cos^3 t, \quad y = \sin^3 t.$$

To plot this, use a `plot` command whose argument is a **list** consisting of the expressions for x and y in terms of t, followed by the range of values of t. In the seed file, this list is assigned the name **astr** to allow later reference to its components. Then, the plot is constructed with a name (suppressing output) to allow later reference and a particular color; and the name is entered to show the plot. These commands are `astr:=[(cos(t))^3,(sin(t))^3,t=0..2*Pi]; A:=plot(astr,color=GREEN): A;` .

To use the **VectorCalculus** package, we build a **Vector** by putting the components of **astr** between **angle brackets** with the instruction `astrVec:=<astr[1..2]>;`. This allows us to find the equation of the tangent line at $t = a$, now using t as the parameter of the line and a to identify the point of tangency, with

```
astrTL:=TangentLine(astrVec,t=a);
```
.

An interesting feature of the curve is that **the segments of all tangent lines between intercepts have the same length**. The seed file contains instructions for using the equation of the line to find the intercepts, build the **Vector** joining them, and find the length of this vector.

The seed file builds a composite `display` with this curve and five of its tangents (at $t = 5\pi/12$, $10\pi/12 = 5\pi/6$, $15\pi/12 = 5\pi/4$, $20\pi/12 = 5\pi/3$, and $25\pi/12$), and the supplementary worksheet continues this example by constructing an **animated plot** of the tangent lines.

In **text**:

- identify the instruction in the seed file that finds the length of the segment of the tangent line between the intercepts; and
- tell which line in your picture belongs to the value $t = 5\pi/4$.

The plot in the supplementary worksheet initially only shows the astroid in the **Maple Plot Structure** `A` . Select this graph to enable a **context bar** at the top of the worksheet, or use a **context menu** when you right-click on the plot to get access to commands for configuring or playing the animation. When you ask to **play** the animation, the tangent lines will appear in the graph.

Problem 7: Surfaces

Build a Maple command to perform the following assignment of an expression to the variable *unu* (a distinctive name chosen to avoid conflict with names that might be used for other things):

$$unu = x(1 - 9xy)e^{-x^2-3y^2}.$$

Before doing anything else, **check that your definition can be evaluated** and **produces numerical answers**. This requires two steps: first say `eval(unu,[x=1,y=1]);` (you may use any numbers, not necessarily $(x, y) = (1, 1)$, but you should use numbers that allow you to compare Maple's answer with the value that you expect); then get a **decimal approximation**, either by typing `evalf(%);`, or by generating an equivalent instruction from a **context menu**. To use the context menu, position your mouse over the output and press the right mouse button to bring up the menu; select **Approximate**, and then the desired number of decimal places. Five or ten places suffice for this check — you don't need any more. If you don't get a number, your function definition is wrong — most likely because you entered `e^(...)` instead of `exp(...)` .

Maple's plotting tools can be used to create an instruction to plot this function including various options. Alternatively, you can add options to a `plot3d(unu);` instruction. The latter approach avoids the clutter of repeating complicate expressions.

As an example of the use of the graphics tools, produce a plot of this surface in the supplementary worksheet with `boxed` axes, and use the mouse to rotate the image to get a **view showing the shape of the surface** and **the labels on the axes** that allows you to see the **maximum value of the function**. The **context bar** at the top of the worksheet will show the values of ϑ (theta) and φ (phi) for this view. Make note of these values and add them as an `orientation` option to the plot instruction. Maple's plotting tools may help in constructing this instruction. The resulting plot should agree with the one you used to choose the view. When you are satisfied that you have suitable values for all options, copy the instruction to your main worksheet and execute it.

Save this result — we will return to this function in a later project where we will use calculus to determine the exact maximum and the point at which it is attained.

End of Lab0