640:251 (Honors Section) FALL 1999

PRACTICE PROBLEMS—EXAM 1

Please note that this is *not* a practice exam; in particular, there are more problems here than will be on the exam. Moreover, although these problems are generally similar to exam problems, it is possible that the exam will contain some problems quite different from any here.

The answers are shown in small type—not guaranteed correct.

1. A particle passes through the point $P(2,1,3)$ at time $t=0$. If the particle is moving with constant velocity $\mathbf{v}=-\mathbf{i}+2\mathbf{j}+\mathbf{k}$, where is the particle at time t?. At what time does the particle pass through the plane with equation $x+2y+3z=20$?

$(2-t, 1+2t, 3+t)$; $t=7/6$

2. Consider the plane through the points $Q(1,3,2)$, $R(2,1,4)$, and $P(1,5,6)$.

(a) Find a normal vector to this plane and an equation for this plane.

(b) Find the angle at Q of the triangle PQR, and find the area of this triangle.

$6\mathbf{i}+2\mathbf{j}-\mathbf{k}$; $6x+2y-z=10$; $\cos^{-1}(2/(3\sqrt{5}))$; $\sqrt{41}$

3. The acceleration of a particle in the plane is $\mathbf{a}=2\mathbf{i}+8\mathbf{j}$.

(a) Find the particle's position as a function of t if $\mathbf{v}(0)$, its velocity at time $t=0$, is $\mathbf{i}-\mathbf{j}$ and $\mathbf{r}(0)$, its position at time $t=0$, is $2\mathbf{i}+3\mathbf{j}$.

(b) Find the speed of the particle at time $t=1$.

(c) Find the curvature κ of the path of the particle at time $t=1$.

(d) Find the normal component of the acceleration of the particle at time $t=1$.

(d) Find the tangential component of the acceleration of the particle at time $t=1$.

$(t^2+t+2)\mathbf{i}+(4t^2-t+3)\mathbf{j}$; $\sqrt{58}$; $10/58^{3/2}$; $10/\sqrt{58}$; $62/\sqrt{58}$

4. Let $f(x,y)=x^2-y$. Sketch the level curves $f=c$ for $c=0$ and $c=1$. Label each curve with the appropriate function value. Calculate a unit tangent vector $\mathbf{T}$ to the curve $f(x,y)=1$ at the point $(2,3)$, the gradient vector $\nabla f(2,3)$, and $\mathbf{T}\cdot\nabla f(2,3)$.

$\mathbf{T}=(\mathbf{i}+4\mathbf{j})/\sqrt{17}$; $\nabla f(2,3)=4\mathbf{i}-\mathbf{j}$; 0

5. Find all first and second order partial derivatives of the function $f(x,y)=x^2(y+1)+y^2e^{2x}$.

$f_x=2x(y+1)+2y^2e^{2x}$; $f_y=x^2+2ye^{2x}$; $f_{xx}=2(y+1)+4y^2e^{2x}$; $f_{xy}=2x+4ye^{2x}$; $f_{yy}=2e^{2x}$

6. Suppose $f(x,y)=xe^y$. Find the equation of the tangent plane to the surface $z=f(x,y)$ at the point $(2,0,2)$. Use differentials and the values of f, f_x and f_y at $(2,0)$ to find an approximate value for $f(2.01,0.02)$. Compare with the computed value of $f(2.01,0.02)$.

$z-2=(x-2)+2y$; 2.05; 2.0506

7. Suppose that $w=f(x,y)$, where f is a function satisfying $f(1,2)=3$, $f_x(1,2)=1$, $f_y(1,2)=-2$, $f_{xx}(1,2)=3$, $f_{xy}(1,2)=2$, and $f_{yy}(1,2)=0$. Suppose further that $x=u+v-1$ and $y=3uv-1$. Find $\dfrac{\partial w}{\partial u}$ and $\dfrac{\partial^2 w}{\partial v\partial u}$ at $u=1, v=1$.

-5; 9

8. Find the derivative of $f(x,y,z)=\cos(xy)+e^{yz}+\ln(zx)$ at the point $P_0(1,0,1/2)$ in the direction of the vector $\mathbf{A}=\mathbf{i}+\mathbf{j}-\sqrt{2}\mathbf{k}$. In what direction does f decrease most rapidly at P_0? Find equations for the tangent plane and normal line to the surface $f(x,y,z)=2-\ln 2$ at P_0.

$3/4-\sqrt{2}$; direction of $-(\mathbf{i}+(1/2)\mathbf{j}+2\mathbf{k})$; $2x+y+4z=4$; $x=1+t, y=t/2, z=(1/2)+2t$

9. Let $f(x,y)=2x^3-6xy+y^2+4y$. Find all critical points of f and classify them as saddle points, local maxima, or local minima.

(1,1) saddle;(2,4) local min

10. Find the absolute maxima and minima of the function $f(x,y)=x^2-xy+y^2-6y$ on the closed triangular plate bounded by the lines $x=0$, $y=8$, and $y=x$.

Min: -12 at (2,4); max: 16 at (0,8), (8,8)