PRACTICE PROBLEMS—EXAM 2

Please note that this is *not* a practice exam; in particular, there are more problems here than will be on the exam. Moreover, although these problems are generally similar to exam problems, it is possible that the exam will contain some problems quite different from any here.

The answers are not guaranteed.

1. Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$. 30 at (1,−2,5); −30 at (−1,2,−5)

2. Let D be the triangular region bounded by the lines $x = 0$, $y = 0$, and $2x + 3y = 1$. Sketch the region D and evaluate $\int\int_D x\,dA$. 1/72

3. Sketch the region of integration and write an equivalent integral with the order of integration reversed for the integral $\int_0^2 \int_{x^2}^{2x} xy\,dy\,dx$. Evaluate the integral in both forms. $\int_0^4 \int_{y/2}^{\sqrt{y}} xy\,dx\,dy = 8/3$

4. Change the rectangular coordinate integral $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-(x^2+y^2)}\,dy\,dx$ into an equivalent polar integral and evaluate the polar integral. $\int_0^{\pi/2} \int_0^2 e^{-r^2} r\,dr\,d\theta = \pi(1-e^{-4})/4$

5. Let D be the region in the first quadrant that lies between the circles $x^2+y^2 = 4$ and $x^2+y^2 = 2x$.
(a) Sketch D and describe D by scanning inequalities in the x, y variables.
(b) Describe D by scanning inequalities in polar coordinates.
(c) Set up the integral $\int\int_D x\,dA$ in polar coordinates and evaluate it. $(16-3\pi)/6$

6. Express the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + 2y + z = 4$ as an iterated triple integral and then evaluate the integral. 16/3

7. Let E denote the region in the first octant that is bounded below by the cone $z = \sqrt{3x^2 + 3y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 9$. Express the volume of E as an iterated triple integral in (i) cylindrical and (ii) spherical coordinates. Then evaluate both integrals. $9\pi(2-\sqrt{3})/4$

8. Find the gradient vector field ∇f for the function $f(x, y) = xy$. Draw the level curves $f = 1$ and $f = -1$ in the region $-2 \le x \le 2$, $-2 \le y \le 2$ and ∇f at several points along the level curves. $y\mathbf{i}+x\mathbf{j}$

9. Evaluate the line integral $\int_C f\,ds$ of $f(x, y) = xy$ along the curve C given by $\mathbf{r}(t) = t\mathbf{i} + t^4\mathbf{j}$ for $0 \le t \le 1$. $(17\sqrt{17}-1)/144$

10. Find the work $\int_C \mathbf{F} \cdot d\mathbf{r}$ done by the force field $\mathbf{F} = y^2\mathbf{i} + x^2\mathbf{j}$ on a particle that moves from $(1, 0)$ to $(-1, 0)$ along the following curves:
(a) C is given parametrically by $\mathbf{r}(t) = \langle 1 - 2t, t(1 - t)\rangle$, $0 \le t \le 1$,
(b) C is part of the parabola $y = x^2 - 1$.
(c) Is $\mathbf{F}$ a conservative force field? Answer this question by two methods. −1/15, −16/15, *no*

11. Determine whether or not the vector field $\mathbf{F}(x, y) = (x + y^2)\mathbf{i} + (2xy + y^2)\mathbf{j}$ is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$. $f(x,y)=xy^2+(1/2)x^2+(1/3)y^3+K$

12. Consider the line integral $\oint_C (2xy^3\,dx + 4x^2y^2\,dy)$ along the curve C: oriented boundary of the 1st quadrant region enclosed by the curves $y = 0$, $x = 1$ and $y = x^3$.

a) Calculate the line integral directly (use separate parametrizations for each part of C and be careful with the orientation). 2/33
b) Use Green's Theorem to evaluate the line integral.