Due 11/9 at Practicum

Mathematics 251 (Honors): Maple Lab 4
TRIPLE INTEGRALS

In this lab, you will use Maple to help visualize the region of integration of a triple integral and compute its value. In particular, you will how the choice of the order of integration affects the complexity of the iterated integral.

Please turn in only the printout of your Maple worksheet, which should include the Maple commands you input and Maple's response. Your worksheet should also include explicit answers to all questions asked and labels on any plots included in your worksheet. These should be inserted by using the **text** feature of Maple. Also use the **text** feature of Maple to include your name and section number on each page of Maple printout (do NOT write any of this material in by hand). Be sure to remove from the worksheet any extraneous material and any errors you have made.

Before you start up Maple, first copy the *seed file* lab4.mws into your directory. To obtain this file, use your web browser (typically Netscape) to go to the Maple part of the web page for this course. Start at the home page of the mathematics department:

http://math.rutgers.edu

and follow the path

Course Materials – Math 251 - Math 251 (Honors Section) – Maple Labs.

There you will find directions for copying the seed file into your home directory. After you start up Maple, import this file into Maple by choosing **Open** from the **File** menu as described in the handout **Instructions for Use of Maple in Mathematics 251**. You should execute the Maple commands that have been placed in this worksheet and to add some of your own.

Important: If you save a worksheet and then reopen it, Maple does not 'remember' the commands in the worksheet, even though the result of the commands appears in the saved worksheet. So each time you use the worksheet, you must 'wake up' Maple by moving the cursor to the beginning of the sheet and then pressing the `Return` key repeatedly until you get to the point that you want to enter new commands in the worksheet.

Integals in Maple

The basic command for integration in Maple is `int( )` For example, to evaluate $\int_0^1 (x+1)\,dx$, type `int(x+1, x=0..1);` To evaluate the double integral

$$\int_0^1 \int_0^{1-x} (xy + x + 1)\,dy\,dx,$$

type `int(int(x*y + x +1, y=0..1-x),x=0..1);` To evaluate the triple integral

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} (xyz + x + 1)\,dz\,dy\,dx,$$

type `int(int(int(x*y*z + x + 1, z=0..1-x-y),y=0..1-x),x=0..1);`

Observe how the nested `int( )` commands correspond to the iterated integrals. However, you must be more careful with the punctuation for Maple (we usually omit the parentheses when we write an iterated integral by hand).

Problem 1. If a solid has mass density $\delta = 1$, then its moment of inertia about the z-axis is given by

$$I_z = \int\int\int (x^2 + y^2)\, dV.$$

We wish to calculate I_z for a solid S defined to be the region between

$$y = 8 - z^2 - 2x^2 \qquad (1) \qquad \text{a paraboloid}$$

and

$$y = z^2 \qquad (2) \qquad \text{a parabolic cylinder}$$

1a. To see what the solid S looks like, plot the two surfaces given above on the same set of axes for $-2 \le x \le 2$, $-\sqrt{8} \le z \le \sqrt{8}$. Rotate the box containing the region to view it from front, side and top angles. You will also see the curve of intersection of the two surfaces more clearly this way.

1b. Since the boundaries of S are given in terms of functions of x and z, it is easiest to calculate I_z by doing the y integration first. What are the upper and lower limits of the y integral (these limits depend on x and z)? Rotate S to get a top view; this will help you see what limits to use.

1c. The next step in finding I_z is to determine the x and z limits of integration. To get these, it may be helpful to view the projection of the intersection of the paraboloid and the parabolic cylinder onto the x-z plane. To get this projection, use the `implicitplot` command, i.e., type `with(plots):` followed by

```
implicitplot(8-z^2-2*x^2=z^2,x=-2..2, z=-sqrt(8)..sqrt(8),
scaling=CONSTRAINED);
```

What type of curve is the intersection of the two surfaces?

1d. Write out (by hand) an explicit expression for I_z as a triple iterated integral (with the limits of integration that you have determined in part 1b. and 1.c). Now translate this expression into Maple using three nested `int` commands with the appropriate limits and evaluate the integral numerically.

1e. Suppose now we want to compute the same integral by doing the z integration first. It is then useful to vizualize the solid with x and y as independent variables. To do this, solve (by hand computation) equations (1) and (2) for z, assigning the names `z1` and `z2` to the Maple expressions for the positive and negative solutions, respectively, of equation (1) and the names `z3` and `z4` to the Maple expressions for the positive and negative solutions, respectively, of equation (2). These four equations each specify a portion of the boundary of S. Plot these four surfaces on a single set of axes over the region $-2 \leq x \leq 2$, $0 \leq y \leq 8$.

It may be useful to view the plot of part (1e) from several viewpoints to do the remaining parts of this assignment.

1f. The difficult part is now to determine the limits of integration. Note that from your plot, it is clear that for some values of x and y the z values inside S range from the bottom of the paraboloid to the top of the paraboloid, while for other values of x and y, the z values inside S range from the bottom of the parabolic cylinder to the top of the parabolic cylinder. Thus, to evaluate I_z we need to write it as the sum of two integrals I_1 and I_2. Determine (and state) the z limits of integration for each of these two integrals.

1g. Now determine the x and y limits of integration for each of the two integrals in part (1f). First note that if you project the graph of the solid S onto the xy plane, you will obtain a region bounded by the line $y = 0$ and the curve $y = f(x)$. Determine (by hand) $f(x)$ and assign the name `f` to the Maple expression for $f(x)$.

1h. To determine the x and y limits of integration of I_1 and I_2, observe that the curve which divides the xy region of integration found in part (1g) into the region used for I_1 and the region used for I_2 is the projection on the xy plane of the intersection of the surfaces $z1$ and $z3$. Determine an equation for this curve in the form $y = g(x)$ and assign the name `g` to the Maple expression for $g(x)$.

1i. To see what the two regions look like, plot the curves $y = 0$, $y = f(x)$, and $y = g(x)$ for $-2 \leq x \leq 2$ on the same set of axes using the `plot` command with the `axes=BOXED` option.

1j. Using these results set up and use Maple to evaluate the sum of the integrals I_1 and I_2. It is easier if you first integrate in z, then in y, and finally in x. As a check, verify that you get the same answer as in part (1d).

BONUS: For (up to) three bonus points, use Maple to compute this integral doing the x integration first (inside). Include all necessary steps (see for example 1e through 1j above) and explain briefly what you are doing at each step.