**Mathematics 251: Lab 2 QUADRIC SURFACES**

Please turn in only the printout of your Maple worksheet. Use the **text** feature of Maple to add a header containing your name. Use the **title** option in all plots to introduce a label that will be kept with the plot when your report is printed.

The worksheet in the *seed file* is divided into **Sections** corresponding to the parts of this project description. It also contains almost all you need for problems 0 and 1; and imitating those instructions should allow you to complete the other parts. You may elaborate on this organization in preparing your report. Also, remove from the worksheet any extraneous material and any errors you have made.

In the past, a large number of graphs in this lab has caused Maple to crash. The lab has been scaled down so this may no longer be a problem. However, be sure to save your work often. If you have difficulty doing a complete report in one worksheet, you may use separate sheets for smaller collections of parts. You will need to use the `restart;` instruction to clear memory, and then reload the `plots` library and (possibly) initialize some other quantities.

A quadric surface is the graph of a second-degree equation in three variables $x$, $y$, and $z$. The most general form of such an equation is:

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0,$$

where $A, B, C, D, E, F, G, H, I$, and $J$ are constants. In this lab, we use Maple to help visualize some types of quadric surfaces that can arise from equations of the above form.

There are three Maple commands that we will use to plot surfaces in three dimensions. These are `plot3d`, `implicitplot3d`, and `display`. The first is always available, but before the second or third of these can be used, one must type the command `with(plots):`. The command `plot3d` may be used to graph a surface when one of the variables can be solved for explicitly in terms of the other two, e.g., $z = f(x, y)$. It can also be used for **parametric** surfaces in which $x$, $y$ and $z$ are given as functions of $u$ and $v$ (or any other names that you prefer). In all cases, ranges of independent variables may be specified. The first variable must have limits that evaluate to constants (if $a$ has been assigned a constant value by an instruction like `a:=3;` , then you may say `x=-a..a` when specifying this part of the range), but the limits for the second variable may depend on the first variable.

The command `implicitplot3d` is used to graph any surface of the form $g(x, y, z) = 0$. This instruction requires **constant** limits for the three variables $x$, $y$ and $z$.

Because the implementation is different, graphs of the same surface drawn using different instructions may not be the same. One theme in this project examines the differences between graphs that are the same *in theory*. Sometimes the difference is minor, but you should notice **all** those differences between plots caused by different implementations.

The command `display` is used to display two previously defined plots at the same time on the same set of axes. Be sure to end a command that **creates a named plot** for later display with a colon: you really don't want the output it generates. (It won't hurt to see it *once*, but it should not be part of the report you submit.) One problem with this use of the `display` command is that you do not see any of the plots until the `display` command is used. You can get around this by first typing an instruction like

```
plot3d(1, y=0..1,z=0..2, color=RED);
```

to view the plot. If it is satisfactory, edit the line by inserting `p1:=` at the beginning and changing the semicolon at the end to a colon, and then execute it again. This names the plot and suppresses all output. (This instruction appears as a comment in the seed file. If you want to use it, remove the comment symbol at the start of the line, but the line should be removed from your final report. )

You may mix plots formed with `plot3d` and `implicitplot3d` in the same `display`, but you may not mix two dimensional plots and three dimensional plots.

**Part 0: Introduction.** Some *Maple* commands are introduced. You are mostly only required to observe, but part (c) asks for an interpretation of the graphs that were drawn.

**(a)** Execute the following sequence of commands.

```
with(plots):
setoptions3d(axes=BOXED,scaling=CONSTRAINED,style=PATCH);
p1:=plot3d(1, y=0..1,z=0..2, color=RED):
p2:=plot3d(2, x=0..1,y=0..x, color=BLUE):
display({p1,p2},title="Planes using simple plot3d");
```

The colors won't appear when printed in black and white, so be sure to identify how different parts were colored when you describe the plots in your answer to part (c). Then,

**(b)** Execute these commands.

```
p3:=plot3d([1,y,z], y=0..1,z=0..2, color=RED):
p4:=plot3d([x,y,2], x=0..1,y=0..x, color=BLUE):
display({p3,p4},title="Planes using parametric plot3d");
```

**(c)** Insert **text** describing the plots and analyzing why such similar commands give such different graphs. Also,

**(d)** Execute these commands.

```
p5:=implicitplot3d(x=1, x=0..1, y=0..1,z=0..2, color=RED):
p6:=implicitplot3d(z=2, x=0..1,y=0..1,z=0..2, color=BLUE):
display({p5,p6},title="Planes using implicitplot3d");
```

Although this instruction only allows constant limits, the planes represented should resemble one of the previous pictures. Answer **in text**: what difference do you notice between this and the previous displays?

**Part 1: An ellipsoid.** An ellipsoid is a quadric surface of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \tag{1}$$

where $a, b,$ and $c$ are positive constants. In this problem, we consider the ellipsoid with $a = 2$, $b = 3$, $c = 5$. First, give *Maple* these values. It will then be possible to use general descriptions although these parameters will have specific interpretations when the worksheet is executed. The seed file contains these assignments. You can use the same worksheet for another example by changing these definitions of $a$, $b$, and $c$ at the top of the worksheet and re-executing.

**(a)** One approach to plotting the surface given by (1) is to solve for $z$, obtaining the two surfaces

$$z = c\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}, \qquad z = -c\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}$$

which correspond to the top and bottom half of the ellipsoid. Each of these surfaces is defined explicitly in the form $z = f(x, y)$ and thus can be graphed using the `plot3d` command. The `plot3d` command requires you to specify a domain over which you wish to plot the graph. For example, `top:=c*sqrt(1-x^2/a^2-y^2/b^2);` is an expression for the top half of the ellipsoid, and `plot3d(top,x=-a..a,y=-b..b);` is a request to plot its graph over a rectangular region containing all points for which the expression `top` can be defined. A **title** has been added in the seed file, but you may replace with one of your own choosing.

It is also possible to construct plots over domains that are not rectangles. The exact domain for which the expression `top` yields real values is

$$-a \le x \le a; \qquad -b\sqrt{1-(x^2/a^2)} \le y \le b\sqrt{1-(x^2/a^2)}.$$

The "seed file" contains instructions for obtaining three variations on this plot. If the plots are the same size (and not too small, please), they should be identical. However, any difference can become exaggerated when translated into instructions to the computer. End this section with a comment on differences that you observe between the plots.

**(b)** The top and bottom of the ellipsoid can be plotted together in the same `plot3d` command. The whole ellipsoid can also be plotted using the `implicitplot3d` command. The "seed file" contains instructions for obtaining plots in both styles. You should also replace the comment in the seed file with a command for plotting both halves using the "elliptical base" style of (a). Note that the name $E1$ is introduced for one of the plots to allow it to be used in later `display` commands. Use titles in the commands that draw the graphs, and adjust the size of the graphs in your worksheet. Finally, describe any differences between these plots.

**(c)** A parametric description of the ellipsoid can be given using a variant on the *polar coordinate* description of the surface of a sphere. The seed file contains instructions to construct a plot called $E2$ of our ellipsoid. Execute this instruction. Since it is the same surface, the result should be similar to previous plots Although both $E1$ and $E2$ show the whole ellipsoid, you should use **text** to identify any differences you notice in the style of presentation of the surface.

**(d)** Use the instructions in the seed file to find the intersection of the ellipsoid $E1$ with the plane $x = 1$. Rotate the plot (and adjust the size) until it clearly shows the curve of intersection. Then, construct a variant to find the intersection of $E1$ with the plane $y = 2$, and a separate plot to show the intersection of $E1$ with the plane $z = 3$. You may also use $E2$ in place of $E1$ if you prefer.

**(e)** The intersection of $E1$ with the plane $y = 2$ can easily be found without *Maple*. Do the algebra to find an equation for the intersection, and use this equation to identify the curve. Use the **text** feature of Maple to insert this information. Does this agree with the plots obtained by *Maple*?

**Part 2: An elliptic hyperboloid.** The surface

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \tag{2}$$

is known as an (elliptic) hyperboloid of one sheet. Continue to use the values $a = 2$, $b = 3$, $c = 5$. This time, we see from (2) that the surface can only be defined for $(x, y)$ *outside* the ellipse $x^2/a^2 + y^2/b^2 = 1$, so a larger domain is needed in order to get a good view of the surface, so we choose

$$-2a \le x \le 2a, \quad -2b \le y \le 2b, \quad -2c \le z \le 2c. \tag{2a}$$

**(a)** Use `implicitplot3d` to obtain a graph of the surface with equation (2) with the bounds given by $(2a)$.

**(b)** Begin an investigation of the intersection of the surface given by (2) with the plane $z = 2x + y + 2$ by constructing a `plot3d` graph of this equation over the rectangle $-2a \leq x \leq 2a$, $-2b \leq y \leq 2b$, including a **color** option so that it is drawn in a single color, and combine it with the result (a) in a single `display`. Try to find a view that shows the shape of this intersection.

**(c)** Find the projection of the curve in (b) into the $xy$-plane by substituting $2x + y + 2$ for $z$ in (2) and constructing an `implicitplot` of the result in a suitable domain in the $xy$-plane.

End of Lab 2