Here are some exercises giving a different point of view on section 15.5. This section is introduced in the lecture on March 8, 2004. The exercises will be due on March 22 (immediately after Spring Break). This will allow your evaluated work to be returned in time for it to be used as part of your preparation for the exam on March 24.

These exercises give some practice with the principles introduced in "Slide 10" from Spring 2003. Be sure that you look at that slide, or print relevant pages (this material is on pages 18 and 19, but other pages may also be useful) from the "Printable file with all slides from lectures 7 through 11" from the web page for that semester.

The centroid of a triangle is known to be the point whose coordinates are the average of the three vertices, and the area can be found by many different methods (one of which is sure to be easy for any given triangle). This means that any linear function of x and y can be integrated over a triangle **with no effort**. The results have been calculated **once** and the result is remembered for its geometric significance. Combining this with Green's theorem, if $P(x, y)$ and $Q(x, y)$ are polynomials of degree at most 2, then

$$\oint_C P(x, y)\, dx + Q(x, y)\, dy$$

can be found if C is a path going once around the perimeter of a triangle – again, with no more work that computing some partial derivatives and using known properties of the triangle. No integrals should be calculated since you have already found them many times.

1. Let C consist of the path consisting of lines from $(0, 0)$ to $(5, 0)$, from $(5, 0)$ to $(1, 3)$, and from $(1, 3)$ to $(0, 0)$. Find

$$\oint_C (x^2 - 3xy + 2y^2 - 4y + 3)\, dx + (2x^2 + xy - y^2 + 5y - 2)\, dy.$$

2. Find

$$\iint_D y\, dA$$

where D is the semicircle inside $x^2 + y^2 = 1$ and above $y = 0$. Either use polar coordinates or find a line integral around the boundary that gives this integral and evaluate the line integral by parameterizing the two curves making up the boundary.

Combine this with what you know about the area of this figure to get the y coordinate of the centroid. (You also know that the centroid is on the y axis, so this gives a geometric description of the location of the centroid of a semicircle.)