Mathematics 251: Lab 1 VECTOR CALCULUS

Please turn in only the printout of your Maple worksheet. Use the **text** feature of Maple to add a header containing your name. The worksheet in the *seed file* is divided into **Sections** corresponding to the parts of this project description. You may elaborate on this organization in preparing your report. Also, remove from the worksheet any extraneous material and any errors you have made.

0. Introduction. The `plots` library is loaded using the `with(plots)` instruction. Instead of using a Maple package to define a large collection of functions, the seed file contains some definitions to implement definitions of this course in a simpler structure called a **list**. Here are the instructions in the introductory section that provide the functions for working this project.

```
with(plots): setoptions3d(scaling=CONSTRAINED);
scalP:=(sc,vec)->map(x->sc*x,vec);
dotP:=(v1,v2)->sum(v1[i]*v2[i],i=1..nops(v1));
crossP:=(v1,v2)->[v1[2]*v2[3]-v1[3]*v2[2],
         v1[3]*v2[1]-v1[1]*v2[3],v1[1]*v2[2]-v1[2]*v2[1]];
proJ:=(a,b)->scalP(dotP(a,b)/dotP(a,a),a);
lenV:=v->sqrt(dotP(v,v)); divV:=(v,c)->scalP(1/c,v);
```

This first line loads the plots library and declares that you want the unit lengths on the three axes to be the same in three dimensional graphs. Then the functions for working with vectors as *lists* are defined. The definitions of these functions should look familiar. Additional names are introduced for the formulas giving projections and length. The multiplication of a vector by the multiplicative inverse of a scalar appears frequently when finding unit vectors, so this has also been given a name. To read these statements, note that they begin with the name name being defined and the assignment symbol `:=`. Next are the names used for the arguments of the function, enclosed in parentheses if there is more than one, and the special symbols `->` that suggests an arrow. Finally, there is an expression for the result in terms of the arguments. The names have been chosen to avoid conflicts with standard parts of Maple. An important feature of these definitions is the use of `map` in the definition of `scalP` to assure that the result is a list in which all components of the `vec` argument have been multiplied by `sc`. Note that individual components of a vector are named by putting the index in square brackets.

1. Testing the definitions. Introduce vectors $\mathbf{v} = \langle 1, 2, -2\rangle$ and $\mathbf{w} = \langle 10, 6, 2\rangle$. Find their cross product and resolve $\mathbf{w}$ into its projection $\mathbf{w}_1$ onto the direction of $\mathbf{v}$ and $\mathbf{w}_2 = \mathbf{w} - \mathbf{w}_1$, which should be perpendicular to $\mathbf{v}$. Then find the length of $\mathbf{v}$ and a unit vector parallel to $\mathbf{v}$. The seed file contains all of these operations and some steps that check that vectors expected to be perpendicular have dot product zero.

Note that addition and subtraction behave in the expected way with this representation of vectors. Multiplication and division by scalars *sometimes* give the expected result, but `scalP` has been defined to *always* give a result in a form that can be used by the other functions defined here. The important construction of a unit vector in the direction of a given vector is illustrated by the instructions

```
lenv:= lenV(v);  # compute the length of v
Uv := divV(v,lenv); # produces a unit vector in the direction of v
```

2. A spacecurve. Consider the curve

$$\mathbf{r}(t) = \left\langle 1 - 2 * (\cos^2 t - \sin^2 t), \cos t, \sin t \right\rangle.$$

Since all components are expressed in terms of $\sin t$ and $\cos t$, $\mathbf{r}(t + 2\pi) = \mathbf{r}(t)$. We introduce a variable r to represent the vector expression, and use the `spacecurve` instruction with an interval of $-\pi \le t \le \pi$ to get a picture of the whole curve. Instructions for obtaining a plot are given here, but you should experiment with different views of the curve, and choose one that seems to suggest the general appearance of the curve in space.

```
r:= [1-2*(cos(t)^2-sin(t)^2),cos(t),sin(t)];
spacecurve(r,t=-Pi..Pi,title="My First Spacecurve",thickness=2);
```

Here, we follow the convention of the textbook of introducing the name `r` for the vector function that draws the curve. This abbreviation can be used in all work with the curve. The **thickness** option makes the graph more visible. Also, the **title** option is used to add a title that will be kept with the graph when your worksheet is divided into pages for printing. We will expect this option to be used to identify all graphs in this course.

3. The first curvature formula. The following formulas for the unit tangent vector T, normal vector N, and the curvature κ can be found in your book:

$$\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}, \qquad \mathbf{N} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}, \qquad \kappa = \frac{|d\mathbf{T}/dt|}{|\mathbf{v}|}.$$

Use Maple to define expressions for each of these, using the letters given above (note that *Maple* recognizes "kappa" as the name of κ), and then simplify these expressions as much as possible. Start by defining **v** to be the derivative of **r**. **Be sure to use the functions defined in the introduction for vector operations to assure that every vector will be represented as a list.** The first versions of some of the results may be complicated. It may be useful to examine these quantities when working through the lab, but appropriate instructions should be used to simplify the results appearing in the printed worksheet.

Be sure you use that correct Maple expressions to obtain these quantities rather than a literal translation of standard mathematical notation. In particular, note that mathematical notation uses $|\mathbf{v}|$ for what this project's version of Maple calls `lenV(v)`.

4. The second curvature formula. Also, the formula

$$\kappa_1 = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3},$$

where **a** is the acceleration vector $\mathbf{a} = d\mathbf{v}/dt$, gives a different computation of the curvature. Use the name `kappa1` for the quantity computed by this formula, and compute a simplified form of κ_1.

5. A third formula for curvature. Another method of computing κ was suggested in Section 13.4. You have already given names to the velocity vector **v** and the acceleration vector **a**. Now $\mathbf{a} - \text{proj}_{\mathbf{v}}\, \mathbf{a}$ is $\kappa(ds/dt)^2\mathbf{N}$. You have the ingredients to compute this using the *proJ* function at the top of the worksheet. This allows κ to be found from the length of this vector and the length of **v**. Compute a quantity κ_2 that describes the value of κ obtained by this approach and simplify it.

6. Conclusion. You have found three expressions that all claim to be κ. Compare the results and answer the following in *text*: Do you see that they are the same? Does Maple?

Although Maple is able to keep track of complicated expressions, you need simple expressions in order to answer these questions. Such simple expressions can be found in this example, although you may need to use the `simplify` instruction to encourage Maple to write expressions that you can read. If the expression does not simplify, or if uses constructions that you do not recognize, it is probably wrong. Be sure that you use the functions defined in the introduction to get the length of a vector an to multiply vectors by scalars.

End of Lab 1