Mathematics 251 Maple Lab 0

Introduction to Maple

Fall 2004

This project This lab contains seven problems intended to introduce you to some of the basic features of Maple and to give you practice preparing a Maple worksheet. Many of the Maple instructions are in the "seed file", but there are several places where you are asked to interpret results. Additional information you need to do use Maple is contained in the handout Instructions for Use of Maple in Mathematics 251.

This lab is for practice only: the lab will be graded using the same standards that will be applied to later labs, but the grade will be ignored in computing your grade for the course, Use this lab to learn how to prepare your Maple worksheet. In particular, be sure to include explicit answers to all questions asked, using the **text** feature of Maple to insert them in the worksheet. Also use the **text** feature of Maple to include your name and section number at the top of the worksheet (do **NOT** write any of this material in by hand). Use the editing capabilities of Maple to remove any extraneous material from the worksheet .

The seed file begins with the line: "Put header here." That line is to be **replaced** with a header that includes your name and any other information requested by your instructor. This is followed by a **Section 0** that contains global settings used in the worksheet. For this lab, there is the line

```
with(plots):with(VectorCalculus):with(LinearAlgebra):
```

that loads the **plots, VectorCalculus, and LinearAlgebra libraries**. Each `with` command ends with a colon to hide the list of all functions in the library that is the normal output of this command. Several lines beginning, "Warning, . . . " may be printed. These are harmless, and may be ignored.

Some of the problems refer to tools in the Student[CalculusI] package. If you want to use these tools, use the Tutor0.mw worksheet available from the course web page.

Problem 1: Pi It is said that the first thing that everyone does when introduced to Maple is to compute many digits of π. Begin by asking for 100 decimal digits with the command `evalf[100](Pi);` (in the seed file).

Various formulas approximate π by something that can be found exactly, with a known bound on the difference. For example, the Maclaurin series for $\arctan(x)$ converges sufficiently quickly that you can be sure that 70 terms suffice to find $\arctan(1/5)$ to 100 decimal places, and 21 terms will give $\arctan(1/239)$ to the same accuracy. This led Machin, in 1706, to compute π to 100 decimal places using the formula

$$\frac{\pi}{4} = 4\arctan\frac{1}{5} - \arctan\frac{1}{239}.$$

Duplicate part of his effort (ignoring the details of how the arctangent is computed) using the Maple commands `MpiOver4:=4*arctan(1/5) - arctan(1/239);` (which shows that Maple accepts unusual names for variables), and `evalf[100](4*MpiOver4);` . Note the use of `*` to denote multiplication.

Although, Maple will do this type of **numerical** computation, its real strength is in **symbolic** work. It can **prove** Machin's formula. To do this, enter `expand(tan(MpiOver4));` . This value is exact: to see how it was done, look at `expand(tan(A+B));` . Follow this with a **text comment** describing how repeated use of this formula can expand `tan(MpiOver4)` . (Space has been left in the seed file for comments requested in this project description.)

Actually, a little more is needed because the $\arctan(x)$ selects the solution of $\tan(u) = x$ with $-\pi/2 < u < \pi/2$, so you also need to check that Machin's value is in this interval. Crude estimates on this value are enough to check this. Instead of doing this, we give a simple example where the arctangent **does not** give the intended value. The seed file contains the lines `expand(tan(2*arctan(2)));` (which returns the value $-4/3$), `V1:=2*arctan(2.);` , and `V2:=arctan(-4./3);` . Use a **text comment** to **explain the difference** between the values of `V1` and `V2`. You should recognize $V1 - V2$. **Include your interpretation** of this value and its significance in your comment.

Problem 2: Evaluation and Simplification

Consider the expression

$$\frac{2x^3 - 7x^2y + 5xy^2}{2x^3 - 7x^2y + 7xy^2 - 5y^3}.$$

Write a command assigning the Maple version of this expression to the **name** `express2`. (The seed file contains a line to be completed and a **text reminder** noting that the line needs to be completed. **Such reminders are to be removed** when you have done what they suggest.)

To test your definition, the seed file contains instructions to evaluate `express2` at $(1, 0)$, $(0, 1)$, $(1, 1)$, and $(1, -1)$. Note how a **list** is used to pass the values of more than one variable to the `eval` command.

Compare the evaluations in the seed file to what you believe to be the values of the function. When you are confident that you are able to evaluate this expression correctly, try to find its value at $(x, y) = (5, 2)$. It should give an error. **This error should be left in your worksheet.**

The line `simplify(express2);` has a **result** that should **look simpler** than the original expression, but you should give the command `express2;` to see what that name represents after executing the `simplify` command. You should find that **it has not changed**. In other words, the `simplify` command **shows you the simplification**, but **it does not modify its argument**. When you give the result to an new name (the seed file contains the line `express2s:=simplify(express2);` to do this), you will have names for both the original and the simplified expression so that they can be compared.

Copy the `eval` commands and change `express` to `express2`. If there was a value for the original expression, you should get the same value for the simplified expression. However, there is now a value at $(x, y) = (5, 2)$.

Add a **text comment** giving what you think is the relation between the original expression and its simplification. This comment may refer to results of this section, or any Maple instructions that you add to the section to support your claims.

Problem 3: Plotting a function

In this part, we use Maple's `plot` command to determine if the function x^x has any local maxima or minima on the interval $0 \le x \le 4$. The seed file contains a line `x^x;` with a reminder giving a pointer to the **Plot Builder**. This is available from the **Plots** submenu at the bottom of the **Context Menu** that appears if you highlight (by selecting with the mouse) the x^x output of this instruction. If you prefer not to use this tool, you may build your own plots using the hints below. It is recommended that you select **Plot Builder** from the submenu since **this will construct a plot command in your worksheet** that will draw the same plot if you execute the worksheet at a later time. The other items, which lead to further menus, construct a `smartplot` instruction that allows many types of modifications, but **does not** remember them between sessions. Whichever method you use, the worksheet **should be edited** to show only the plots enumerated later in this section and the instructions that draw them **in increasing level of detail**.

The first plot should show $y = x^x$ on the interval $0 < x < 4$. From the first dialog box of **Plot Builder**, you should select **2-D plot** to get a menu of available options. Some lines should already be filled in: the function `x->x^x` in the top line; the label x (with horizontal orientation) for the domain; and several technical options on the lower part of the page. Begin by adjusting the domain to be from 0 to 4 and adding a **Title** such as "My first plot". You may also specify that the **Range** should have the label y (you may leave the orientation as horizontal, since this refers to the **label** on the axis and not the axis itself). When you close the Plot Builder by selecting **Plot** at the bottom of the box, an instruction to draw the plot will be written and executed at that point in the worksheet. With the choices described here, it should read `plot(x^x,x = 0 ..  4,labels = ["x", "y"],title = "My First plot");`.

One way to get a more useful graph, is to restrict the second coordinate. For example, setting the range to be from 0 to 10 and giving a suitable title leads to an instruction `plot(x^x,x = 0 ..  4,view = [DEFAULT, 0 ..  10],labels = ["x", "y"],title = "Limited range");`. (If you Construct the `plot` command by hand, you can avoid the awkward **view** and **labels** options using the instruction `plot(x^x,x=0..4,y=0..10,title = "Limited range");` — the **title** option should always be used).

To get a plot that **zooms in on** the critical point, **choose a smaller domain** (you may remove the restriction on the range if the domain is small enough). Draw several graphs to **zoom in on** the critical point allowing you to **identify the nature** (local maximum or minimum) and **estimate the coordinates** of the critical point. Remove all but three graphs: (1) the **original** plot with $0 < x < 4$; (2) the **limited range plot** with $0 < x < 4$ and $0 < y < 10$; and (3) one graph showing a **small neighborhood** of the critical point.

Add a text comment interpreting your third graph. The comment **should indicate** whether the critical point is a local minimum or a local maximum, and **give its coordinates** to **two decimal places**. (By using Calculus, Maple can find the location of the critical point **exactly**, but that is not part of this project. One method is to use the `ExtremePoints` function in the **Student[Calculus1] package**, that you can load by including a line `with(Student[Calculus1]):` — note the colon — in a worksheet. Since this is not a part of the course project, such excursions should be done on a separate worksheet. The worksheet that you submit should contain only instructions and output relevant requested in this project description.

Problem 4: Derivatives

Find the first, second, and third derivatives of the expression e^{x^2}.

First, write the **Maple input** for this expression (using the `exp` function, **not** the `^` operator, and **assign** the result to the name `Exsq`. Then repeatedly use Maple's `diff` command to introduce new named expressions for the higher derivatives. The names `Exsq1`, `Exsq2`, and `Exsq3` are suggested because they are descriptive, but you may use any names.

Collect these into a **list** — consult **Maple Help**, if necessary, to see how to do this — and assign this list to a named variable.

The values of these derivatives at $x = 0$ are related to the series for this function by **Taylor's theorem**. Use the `eval` command applied to the **name of the list** that you just constructed to find a list of these values. Some of the terms should be zero. Do you think this is the beginning of a pattern? Use a **text comment** to suggest a pattern and identify other things that might be done to test for a pattern.

Problem 5: Integrals

Use Maple's `int` command to find the **indefinite integral** $\int x^2 \cos(x)\, dx$ and the **definite integral** $\int_0^\pi x^2 \cos(x)\, dx$. If you want to step through this integration, use the **Tutor0.mw** worksheet from the course web page.

When you were first learning **techniques of integration**, you were discouraged from attempting to find

$$\int e^{-x^2}\, dx.$$

Since the integrand is continuous, **the integral certainly exists**, but it **cannot be expressed in terms of the familiar functions of calculus**. Maple knows how to find this integral, because it knows about the **new function that was invented** to express this integral. The seed file contains the instructions `int( exp(-x^2), x );` for finding the indefinite integral, and `int( exp(-x^2), x =-infinity .. infinity );` to illustrate how certain **improper integrals** are written in Maple.

Note that Maple doesn't write the $+C$ that you were told to use to indicate that you were finished computing an integral. If you want to define a function as an integral, it is necessary to specify the value of the function at some **base point** — we have a preference for functions that satisfy $f(0) = 0$, although there are exceptions.

End this section with a **text comment** giving: the **name** that Maple uses for the function used to evaluate this integral; the **value** of that function at zero. Your answer should be supported by a Maple computation evaluating this function at zero. The integral we studied is **an unusual multiple** of the new function. Your comment should also consider what property of the function makes it advantageous to use this definition instead of our integral.

Problem 6: Parametric curves

Maple's `plot` command can also be used to plot parametric equations. The **astroid** is defined by

$$x = \cos^3 t, \quad y = \sin^3 t.$$

To plot this, use a `plot` command whose argument is a **list** consisting of the expressions for x and y in terms of t, followed by the range of values of t. In the seed file, this list is assigned the name **astr** to allow later reference to its components. Then, the plot is constructed with a name (suppressing output) to allow later reference and a particular color; and the name is entered to show the plot. These commands are `astr:=[(cos(t))^3,(sin(t))^3,t=0..2*Pi]; A:=plot(astr,color=GREEN): A;` .

To use the **VectorCalculus** package, we build a **Vector** by putting the components of **astr** between **angle brackets** with the instruction `astrVec:=<astr[1..2]>;`. This allows us to find the equation of the tangent line at $t = a$, now using t as the parameter of the line and a to identify the point of tangency, with

`astrTL:=TangentLine(astrVec,t=a);` .

An interesting feature of the curve is that **the segments of all tangent lines between intercepts have the same length**. The seed file contains instructions for using the equation of the line to find the intercepts, build the **Vector** joining them, and find the length of this vector. They will not be reproduced here, but we note that finding the length required the `Norm` command from the **LinearAlgebra** package. The full simplification of the expression for the length requires knowing that we are not interested in complex values of a. The **assuming** phrase takes care of that. This example is a trickier use of Maple than you will be required to construct on your own, but you are encouraged to use it as a model for applying Maple to studying tangent lines that appear in any of your studies.

To return to graphing, the command `LS:=[astrTL[1],astrTL[2],t=t0..t1];` builds a list that can be given to a `plot` command to draw a tangent line for a **specified value** of a. For example, `TL1:=plot(eval(LS,a=Pi/4),color=RED):` constructs a plot of the tangent line at a point in the first quadrant. A `display` command is used to combine the two plots constructed so far.

You should **follow this model** for $a = 4\pi/3$, $7\pi/6$, and $15\pi/16$: **build commands** with **individual names** to draw the lines in **your choice of colors**; then combine all plots in a single `display` command.

A **Diversion** section has been added to the seed file. The section is initially **hidden**. To **show** it, click on the + sign beside the word "Diversion". This will expose the section and **negate** the sign that you clicked. Clicking this − sign will hide the section again. This special section is just for your amusement, and not part of the graded project, so it **should be hidden when the worksheet is printed**. It contains instructions for constructing an **animated plot**. This plot initially only shows the astroid in the **Maple Plot Structure** `A`. Select this graph to enable a **context bar** at the top of the worksheet, or use a **context menu** when you right-click on the plot to get access to commands for configuring or playing the animation. When you ask to **play** the animation, the tangent lines will appear in the graph.

Problem 7: Surfaces

Build a Maple command to perform the following assignment of an expression to the variable z:

$$z = y(1 - 8xy)e^{-2x^2-y^2}.$$

Before doing anything else, **check that your definition can be evaluated** and **produces numerical answers**. This requires two steps: first say `eval(z,[x=1,y=1]);` (you may use any numbers, not necessarily $(x, y) = (1, 1)$, but you should use numbers that allow you to compare Maple's answer with the value that you expect); then get a **decimal approximation**, either by typing `evalf(%);`, or by generating an equivalent instruction from a **context menu**. To use the context menu, position your mouse over the output and press the right button to bring up the menu; select **Approximate**, and then the desired number of decimal places. Five or ten places suffice for this check — you don't need any more. If you don't get a number, your function definition is wrong — most likely because you entered `e^(...)` instead of `exp(...)`.

Now you can use **Plot Builder** on the output of `z;` to construct a plot of $z = y(1 - 8xy)e^{-2x^2-y^2}$ over the range $-3 \le x \le 3$, $-3 \le y \le 3$. (Use of this tool is not required, but it is an easy way to set all the options that you need and have Maple remember them as you refine your plot.) Select **Plots**, then **Plot Builder** from the context menu. You will be given a choice of types of plots from which you should choose **3-D plot** (it should be pre-selected). The function to be plotted will already be entered into the **Plot Builder Dialog** that opens after choosing the type of plot. On your first use of the tool, you will need to provide the **domain** and a **title**, but Maple will remember these choices whenever you launch the Plot Builder in the same session. You should also name the z axis without specifying a range and choose **boxed** from the options in the **Axes** section.

You should adjust this plot to get a **view showing the shape of the surface** and **the labels on the axes** to allow you to see the **maximum value of the function**. The view should be selected to The **context bar** at the top of the worksheet will show the values of ϑ (theta) and φ (phi) for this view. Make note of these values. Then return to the Plot Builder and enter these values into the dialog. The resulting plot should agree with the one you used to choose the view.

Edit the worksheet to remove all but the last plot that you obtained. Then, add a **text comment** giving your estimate (to about one decimal place) of the maximum value of the function. Save this result — we will return to this function in a later project where we will use calculus to determine the exact maximum and the point at which it is attained.

End of Lab0