Distance formula If d is distance from $P_0(x_0, y_0, z_0)$ to $P_1(x_1, y_1, z_1)$, then

$$d^2 = (x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2.$$

Exercise 12.1.9a Are the points

$$A(5, 1, 3) \quad B(7, 9, -1) \quad C(1, -15, 11)$$

collinear?

Method First compute distances.

Exercise 12.1.11 Find equation of sphere with center $(0, 1, -1)$ and radius 4. What is intersection with yz-plane?

Method Use distance formula.

Exercise 12.1.15 Identify sphere

$$x^2 + y^2 + z^2 + 2x + 8y - 4z = 28.$$

Method Complete the square.

Exercise 12.1.40 Consider the points P such that the distance from P to $A(-1, 5, 3)$ is twice the distance from P to $B(6, 2, -2)$. Show that the set of all such points is a sphere and find its center and radius.

Method Let P be (x, y, z) and express given information as an equation.

Vectors (in space) A triple $\mathbf{v} = \langle a, b, c\rangle$ representing the **displacement** from point (x, y, z) to $(x + a, y + b, z + c)$. Vectors may be **added** by adding corresponding entries, or **scaled** by multiplying each entry by same number. The segments illustrating a vector are all **parallel**, **of equal length**, and **similarly oriented**. The rule

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$

is illustrated by a **parallelogram**; scaling is illustrated by **similar triangles**. The **length** of a vector **v** (denoted $|\mathbf{v}|$) is given by the **distance formula**. The **direction** of a vector **v**, is a **positive multiple** of **v** of length 1. The **zero vector** $\langle 0, 0, 0\rangle$ has **no** direction (although we sometimes consider it to have **all** directions). Vectors are often written as a sum of multiples of the special vectors

$$\mathbf{i} = \langle 1, 0, 0\rangle$$
$$\mathbf{j} = \langle 0, 1, 0\rangle$$
$$\mathbf{k} = \langle 0, 0, 1\rangle$$

Exercise 12.2.20 Find $|\mathbf{a}|$, $\mathbf{a}+\mathbf{b}$, $\mathbf{a}-\mathbf{b}$, $2\mathbf{a}$, and $3\mathbf{a}+4\mathbf{b}$ for

$$\mathbf{a} = \langle -3, -4, -1\rangle$$
$$\mathbf{b} = \langle 6, 2, -3\rangle$$

Method Use definitions.

Exercise 12.2.22 Same instructions as 12.2.20, with

$$\mathbf{a} = 3\mathbf{i} - 2\mathbf{k}$$
$$\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$$

Method Combine and simplify.

Exercise 12.2.26 Find a unit vector in the same direction as $\langle 1, -2, 3\rangle$.