Math 251:10–12 — Spring 1999
MW6 CHM-106
Prof. Bumby

This section on the Web. By now you should have discovered the home page for this section of 251. Everything distributed in class is available there along with copies of slides and initial files for the *Maple* labs. Slides are originally posted as individual lectures, but they may be combined later.

Previously, there was a defect in the files that caused missing letters when a running Acrobat reader was called to display a new file. I think I have fixed this. Some reports of font problems in *Maple* have been reported. If you notice anything strange, let me know — it may not be your fault. There may be other hidden features (i.e., bugs) in posted files. Please report any difficulties.

Overview of part 3 of the course: March 03–29. Some material from the beginning of Chapter 14 will be introduced before starting Chapter 13. Green's theorem is so closely tied to the evaluation of double integrals that the two topics should be discussed together. If the material was covered in the order used in the textbook, this simplifying principle could get lost in a jumble of new theorems. The rapid pace of the lectures before Spring Break is designed to allow the lecture of March 22 to concentrate on Green's theorem. The material on surface area and integrals introduced on March 24 will be treated more as a source of double integrals to which the earlier our techniques apply than a new topic with new formulas.

The integrals with respect to arc length introduced at the beginning of Section 14.2 have limited applications, and are a distraction from the main theme of the course. While the viewpoint of single-variable calculus seems to lead in that direction, it is the integral of vector fields given in Definition 13 on p. 884 that leads to better theory. From the mathematical viewpoint, you don't have calculus without a fundamental theorem relating integrals and derivatives, and it is the integral of vector fields that leads to such a theorem. However, there is an important new ingredient — only *some* integrals can be evaluated by inverting a form of differentiation, but those integrals are easily recognized. The calculation that recognizes the integrals that are independent of path will appear again when we introduce the curl of a vector field. The integrals of vector fields are also significant from the viewpoint of physics. The physical concept of *work* is expressed naturally as this type of line integral, and the integrals that occupy most of our attention are those for which the force is a function only of position. Since time is limited, scalar integrals with respect to arc length will be ignored (except for this statement that they will be ignored).

Similarly, the slow introduction of double integrals at the beginning of chapter 13 creates a false belief that constant limits of integration might appear in real problems. Be warned: I consider rectangles too dull to use in examination problems. When you are using `plot3d` to describe a surface with equation $z = f(x, y)$ in *Maple*, you are able to specify an arbitrary domain in the xy-plane over which to graph the function. The same rules used to specify such domains are used in multiple integrals. The outer integral, which represents the last step in the computation, must have constant limits of integration to give a numerical result; but the variable of integration in this integral may appear in the limits of integration of any *contained* integral. There is a restriction to rectangular regions when *Maple* is constructing implicit plots, since these limits mainly serve to limit the view of the graph.

Green's theorem also forces one to consider orientation of integrals over two-dimensional regions. At the beginning of Chapter 13, a geometric analysis that hides difficulties with orientation is used to determine the effect of interchanging order of integration. When these integrals are interpreted via Green's theorem, orientation is *visible* as the difference between whether the region is to our left or to our right as we traverse its boundary. However, you should be able to rely on geometry for the problems met in this part of the course.

The exercises from Section 14.4 are all *tricks*. The proof of Green's theorem is based on interpreting the method for evaluating double integrals as line integrals around the boundary. The exercises that "use" Green's theorem to evaluate line integrals by turning them into double integrals are really doing something else. Whatever you do to evaluate the double integral will give you an equivalent line integral around the boundary. The difference between this integral and the given one turns out to be the integral of a conservative vector field over the boundary.

Polar coordinates will make a brief appearance, but they are a good source of exercises since some cumbersome problems in rectangular coordinates become very simple when expressed in polar coordinates.

Maple Labs. By Monday, March 08, you will have had Lab 2 for two weeks. You should aim to finish it by that date, in order to be sure of finishing before Spring Break. Lab 3 will be distributed on March 08. The fourth lab will be distributed on March 29.

Details. Here are the topics to be covered in lectures on Chapter 12 leading to the exam on March 01.

Date	Section	Page	Problems
March 03	14.1	876	18, 20, 22.
	14.2	886	6, 8, 18, 20.
	14.3	895	4, 6, 12, 20, 22.
March 08	13.1	817	NONE
	13.2	822	NONE
	13.3	830	2, 6, 8, 10, 12, 16, 18.
March 10	13.4	836	2, 4, 6, 12, 18, 22, 26, 28.
March 22	14.4	902	2, 4, 8, 12, 14.
March 24	13.6	844	2, 4, 10.