Math 251:10–12 — Spring 1999
MW6 CHM-106
Prof. Bumby

This section on the Web. The home page of the course is updated regularly. All handouts will appear there, although the paper copies distributed in class continue to be considered the official release.

General announcements. No further paper handouts are planned, so the schedule through the end of the term will be given here. All new material in the course will be introduced in the next 5 lectures, leading to an exam on **Monday, April 19**. This will be followed by four lectures: Wednesday, April 21; Monday, April 26; Wednesday, April 28; and Friday (following a Monday schedule, so that office hours will be on the Douglass Campus before lecture instead of on Busch Campus), April 30. These four lectures will serve to review the details, as they appeared on the class exams. The plan is to go over the class exams in order, one per lecture. Questions, or requests for solutions of other problems should follow this schedule. Such requests may either be made in advance to allow for the possibility that a slide will be prepared on that topic, or raised in lecture. In additions, connections between topics that appeared in different parts of the course will be mentioned. The final exam is scheduled for the regular lecture room, CHM-106, on **Wednesday, May 12, 8–11 AM**. I will be available in Hill 438 (Busch Campus) during the time between the end of classes and the final exam, but will only have *scheduled* office hours from 2–5 PM on Monday, May 10 and Tuesday, May 11.

Overview of part 4 of the course:March 31–April 19. In this part of the course, the integral calculus will be extended to three dimensions. As in dimensions one and two, the physical applications of the integral depend on having a definition in terms of Riemann sums, and calculus is possible because the Riemann sums converge when the function is continuous except on a lower dimensional set. Since all of the difficulties were met in defining two dimensional integrals, we will skip the details. Evaluation of an integral is done by writing it as an **iterated integral**. Most of the examples will be *simple regions*, like those met in the discussion of Green's theorem. That is, for each choice of two of the coordinates, the values of the third coordinate of points in the region is always an interval, and the region for which this interval is nonempty is a simple region in the plane. This leads to an outer integral with respect to one of the coordinate variables with constant limits of integration, followed by an integral with respect to a second coordinate variable whose limits may depend on the variable of the outer integral, followed by an integral with respect to the remaining coordinate whose limits may depend on the first two coordinates. The expression $dz\,dy\,dx$ written at the end of an expression for such an integral has the variables arranged so that the last term is paired with the first integral sign. In applications, one often starts with a geometric description of the region and writes dV in place of $dz\,dy\,dx$, since the volume of the region is given by integrating the constant 1 over the region. This special case has appeared in sections on double integrals, since the inner integral

$$\int_0^{f(x,y)} dz = f(x, y),$$

and one can give a direct justification that integrating this over a region in the xy plane gives the volume under the graph of $z = f(x, y)$. The advantage of using triple integrals is that expressions for moments are easily justified using Riemann sums, leading to formulas that have the same appearance as the formula for the volume. By spending so little time on triple integrals, I hope to convince you that this topic contains nothing new.

Section 11.10 gives the basis of *cylindrical* and *spherical* coordinates in space. These coordinate systems are introduced here to allow simpler treatment of integrals having a form of symmetry that allows a simpler description in these coordinate systems. You will be tempted to attach more importance to the formulas expressing rectangular coordinates in terms of spherical coordinates on p. 731 than they deserve. To discourage this, no exercises are assigned that call for directly converting between rectangular and spherical coordinates. It is almost always easier to relate these coordinate systems via cylindrical coordinates. The relation between rectangular and cylindrical coordinates is just the relation between rectangular x and y coordinates and polar r and θ coordinates in a horizontal plane, with a z coordinate having the same interpretation in each system. In relating cylindrical and spherical coordinates, the common coordinate is θ, and each fixed value of θ determines an *axial* plane containing the z-axis. In such a plane, the r and z of cylindrical coordinates are a rectangular coordinate system, and the ρ and ϕ of spherical coordinates are polar coordinates with initial direction $\phi = 0$ being the positive z axis (through the North Pole). In geographical terms, θ is longitude and ϕ is co-latitude. It turns out that this choice leads to simpler formulas than you would get if you used the angle from the equator (i.e., latitude) as your coordinate.

Section 14.6 introduces the idea of parameterizing a surface by giving the coordinates x, y, and z of $\mathbb{R}^3$ as functions of *two* coordinates u and v. In the important special case in which the surface is the graph of a function $z = f(x, y)$, we get a parametric description

$$x = u \quad y = v \quad z = f(u, v).$$

Introducing new names for the parameters allows an interpretation in which u and v are considered as coordinates on the surface, while x, y, and z are used only as coordinates in $\mathbb{R}^3$. Another example of a parametrized surface is the description

$$x = \sin\phi\cos\theta \quad y = \sin\phi\sin\theta \quad z = \cos\phi$$

of the unit sphere via spherical coordinates. Our first treatment of this section will be light, but we will return to it as needed when studying surface integrals.

Here is a detailed outline of the work associated with individual lectures.

Date	Section	Page	Problems
March 31	13.7	852	6, 8, 12, 18
	11.10	733	2, 8, 26, 32, 34, 38, 44
	13.8	858	6, 8, 16, 34
April 05	14.5	909	2, 6, 10, 20
	14.6	917	NONE
	14.7	929	6, 8, 16, 18
April 07	14.8	935	2, 4, 8, 10
April 12	14.9	940	4, 6, 12
April 14	13.9	866	2, 6, 12, 14